Modern Physics: Class Exam II

17 April 2020

Name: ______________________________ **Total:** **/40**

Instructions

- There are 8 questions on 7 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$c = 3.0 \times 10^8\,\mathrm{m/s} \qquad h = 6.63 \times 10^{-34}\,\mathrm{Js} \qquad k_B = 1.38 \times 10^{-23}\,\mathrm{J/K} \qquad 1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$$

$$m_{\text{electron}} = 9.11 \times 10^{-31}\,\mathrm{kg} \qquad m_{\text{proton}} = 1.67 \times 10^{-27}\,\mathrm{kg}$$

$$\int x^n\,dx = (n+1)x^{n+1}$$

$$\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int x\sin^2(ax)\,dx = \frac{x^2}{4} - \frac{x\sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2}$$

$$\int x\sin(ax)\sin(bx)\,dx = \frac{\cos\left((a-b)x\right)}{2(a-b)^2} + x\frac{\sin\left((a-b)x\right)}{2(a-b)} - \frac{\cos\left((a+b)x\right)}{2(a+b)^2} - x\frac{\sin\left((a+b)x\right)}{2(a+b)} \qquad \text{if} \quad a \neq b$$

$$\int x^2\sin^2(ax)\,dx = \frac{x^3}{6} - \frac{x^2}{4a}\sin(2ax) - \frac{x}{4a^2}\cos(2ax) + \frac{1}{8a^3}\sin(2ax)$$

$$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} xe^{-ax^2}\,dx = 0$$

$$\int_{-\infty}^{\infty} x^2e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}}\frac{1}{2a}$$

Question 1

A random number generator is configured so that it produces **one of four numbers.** These numbers are: $2, 3, 4$ and 5. The probabilities with which the random number generator produces each number are all the same. Determine the mean of the numbers produced by this random number generator.

/3

Question 2

An alpha particle, with mass 6.64×10^{-27} kg is trapped inside a one dimensional infinite well with width 7.0×10^{-15} m. Consider motion along the x axis only.

a) Determine the range of likely momenta for the alpha particle.

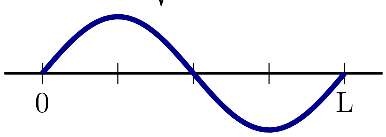
b) Someone claims that it is impossible that the speed of the alpha particle could be larger than 8000 m/s. Is this claim true or false? Explain your answer?

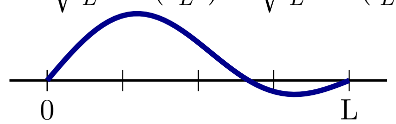
/3

Question 3

Consider a particle in a one dimensional infinite well. The particle is restricted to the range $0 \leqslant x \leqslant L$. At one instant you are told that the wavefunction, $\psi(x)$, could be one of the two displayed below.

$$\psi_1 = \sqrt{\frac{2}{L}}\,\sin\left(\frac{\pi x}{L}\right) \qquad \psi_3 = \sqrt{\frac{2}{L}}\,\sin\left(\frac{2\pi x}{L}\right) + \sqrt{\frac{2}{L}}\,\sin\left(\frac{\pi x}{L}\right)$$





Suppose that you are given one or many particles, each of which could be in one of these two states (and no others). You are guaranteed that **all of the particles are in the same state** but do not know which it is. You have a measuring device which can only determine whether the particle is in the left half of the well ($0 \leqslant x \leqslant L/2$) or the right half of the well ($L/2 \leqslant x \leqslant L$).

a) If you are only given a single particle could you use the measuring device described above to determine in which state it is? Explain your answer.

b) If you are given one million copies of a single particle (all in the same state) could you use the measuring device described above to determine (with some probability) in which state it is? Explain your answer.

/4

Question 4

A quantum harmonic oscillator has potential

$$U(x) = \frac{1}{2} m\omega_0^2 x^2.$$

Consider the following as possible energy eigenstates:

$$\phi(x) = Ax^2$$
$$\psi(x) = Be^{-ax^2}$$

where A, B and a are constants. Check by direct substitution into the time-independent Schrödinger equation whether **each** of these is a possible energy eigenstate/stationary state. If it is, determine the energy associated with the state.

/7

Question 5

An electron is trapped in a one-dimensional finite well. The largest wavelength emitted by the electron is 750 nm.

a) The largest wavelength emitted corresponds to a transition between two states. Which two states are these?

b) Determine the width of the well.

c) Determine the third smallest wavelength emitted by the electron.

/8

Question 6

A particle is in a one dimensional infinite well with potential

$$U(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{otherwise.} \end{cases}$$

At one particular instant the state of the particle is described by the wavefunction

$$\psi(x) = \begin{cases} \sqrt{\dfrac{30}{L^5}}\, x(x-L) & 0 < x < L \\ 0 & \text{otherwise.} \end{cases}$$

Determine the expectation value, $\langle x \rangle$, for position measurements for this particle.

/6

Question 7

A particle in an infinite well is in its lowest energy state. Determine the expectation value of momentum, $\langle p \rangle$ for this particle.

/4

Question 8

A proton with kinetic energy $10\,\text{eV}$ is incident on a barrier with energy $80\,\text{eV}$. For a particular barrier width, the transmission coefficient for particles is 0.00010. Suppose that the barrier width is tripled (three times what it had been previously). Determine the transmission coefficient for the new barrier width. In both cases the wide barrier approximation is applicable.

/5