Modern Physics: Class Exam I

21 February 2020

Name: ______________________ **Total:** /40

Instructions

- There are 7 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$c = 3.0 \times 10^8\,\text{m/s}$ $h = 6.63 \times 10^{-34}\,\text{Js}$ $k_B = 1.38 \times 10^{-23}\,\text{J/K}$ $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$

$m_{\text{electron}} = 9.11 \times 10^{-31}\,\text{kg}$ $m_{\text{proton}} = 1.67 \times 10^{-27}\,\text{kg}$

Question 1

A Helium Neon laser produces light with wavelength 632.8 nm and power 0.010 W.

a) Determine the number of photons produced every second by the laser.

Laser produces 0.010J every second.

Each photon $E_{ph} = hc/\lambda = \frac{6.63\times10^{-34}\,J\cdot s \times 3\times10^8\,m/s}{632.8\times10^{-9}\,m} = 3.14\times10^{-19}J$ +1 +1

+1 number photons × E_{ph} = 0.010J ⇒ number of photons = $\frac{0.010\,J}{3.14\times10^{-19}\,J} = 3.19\times10^{16}$

b) The laser light is incident on a metal with work function 0.90 eV. Determine the (maximum possible) number of electrons ejected from the metal in every second assuming that all of the laser light is incident on the metal.

+1 Requires energy 0.90eV = 0.90eV × 1.6×10^{-19}J/eV = 1.44×10^{-19}J

+1 to remove an electron. Each photon has this so each removes one electron

⇒ 3.19×10^{16} electrons per second

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$$\lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta)$$

always ≥ 0 so $\lambda_2 > \lambda_1$

then smaller m $\Rightarrow$ larger $\lambda_2 - \lambda_1$ $\Rightarrow$ larger λ_2

Question 2

In the Compton effect experiment X-rays are scattered off of various subatomic particles. Suppose that X-rays with one particular wavelength λ_1 are incident on *either* electrons *or* protons. In *both* cases the scattered X-rays are observed at 30°. Which of the following (choose one) is true?

i) The wavelength of the scattered X-rays is exactly λ_1 in both cases.

ii) The wavelength of the scattered X-rays is less than λ_1 in both cases. The wavelength of the X-rays scattered off the electrons is *smaller* than the wavelength of the X-rays scattered off the protons.

iii) The wavelength of the scattered X-rays is less than λ_1 in both cases. The wavelength of the X-rays scattered off the electrons is *larger* than the wavelength of the X-rays scattered off the protons.

iv) The wavelength of the scattered X-rays is more than λ_1 in both cases. The wavelength of the X-rays scattered off the electrons is *smaller* than the wavelength of the X-rays scattered off the protons.

v) The wavelength of the scattered X-rays is more than λ_1 in both cases. The wavelength of the X-rays scattered off the electrons is *larger* than the wavelength of the X-rays scattered off the protons.

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Question 3

X-rays are produced by colliding an electron with a metal. Determine the minimum velocity of the electrons prior to collision such that the X-rays produced have wavelength 0.0850 nm.

The energy of the electron is converted into photon energy. So

$$\Delta K + E_{ph} = 0$$

$$K_f - K_i = -E_{ph} \Rightarrow K_i = E_{ph}$$

($K_f = 0$)

$$\Rightarrow \frac{1}{2}mv^2 = \frac{hc}{\lambda}$$

$$\Rightarrow v^2 = \frac{2hc}{m\lambda}$$

$$\Rightarrow v = \sqrt{\frac{2hc}{m\lambda}} = \sqrt{\frac{2 \times 6.63\times10^{-34}\,\text{Js} \times 3.0\times10^{8}\,\text{m/s}}{9.11\times10^{-31}\,\text{kg} \times 0.085\times10^{-9}\,\text{m}}}$$

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$$= 7.16\times10^{7}\,\text{m/s}$$

Question 4

Two sources produce waves traveling along the $+x$ direction. The expressions for the waves are

$$\psi_1 = Ae^{i(kx-\omega t)}$$
$$\psi_2 = Ae^{i(kx'-\omega t)}$$

where $x' = x + \Delta x$ and A is real.

a) Determine an expression, using complex exponentials, of the superposition, ψ (i.e. total combination of the two waves).

$$\psi = \psi_1 + \psi_2 = A\left[e^{i(kx-\omega t)} + e^{i(kx'-\omega t)}\right]$$

+1

$$= A\left[e^{i(kx-\omega t)} + e^{i(kx + k\Delta x - \omega t)}\right]$$

b) Using the rule that the intensity of the wave is $I = |\psi|^2$, determine an expression for the intensity of the total wave. The expression for the intensity must only involve real quantities and no imaginary quantities must appear (numbers, constants and functions).

+5

$$\psi = A\, e^{i(kx-\omega t)}\left[1 + e^{ik\Delta x}\right]$$

$$|\psi|^2 = \left| A\, e^{i(kx-\omega t)}\left[1 + e^{ik\Delta x}\right]\right|^2$$

$$= \underbrace{|A|^2}_{A^2}\underbrace{|e^{i(kx-\omega t)}|^2}_{1}\left|\left[1 + e^{ik\Delta x}\right]\right|^2$$

$$= A^2\left(1+e^{ik\Delta x}\right)\left(1+e^{-ik\Delta x}\right)$$

$$= A^2\left(2 + \underbrace{e^{ik\Delta x} + e^{-ik\Delta x}}_{2\cos k\Delta x}\right) \Rightarrow I = A^2 2(1+\cos(k\Delta x))$$

Question 4 continued …

c) Use the expression from the previous part to determine all possible values for Δx such that intensity of the superposition is a maximum.

need $\cos(k\Delta x) = 1$ +1

$\Rightarrow k\Delta x = 2\pi m \quad m = 0, \pm 1, \pm 2, \ldots$

$\Rightarrow \Delta x = \frac{2\pi}{k} m \quad m = 0, \pm 1, \pm 2, \ldots$ +1

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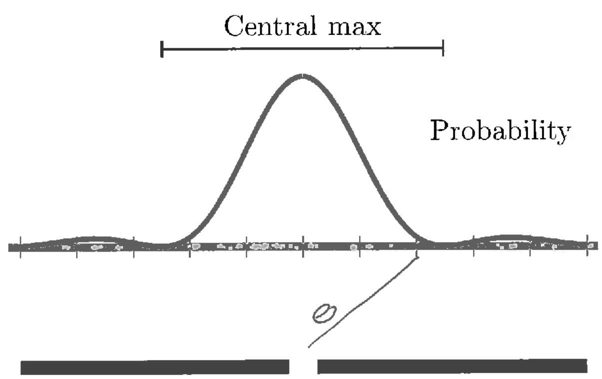
Question 5

Protons are fired toward a single slit. The probability with which the protons arrive at a screen is illustrated. Suppose that, in a separate experiment, the electrons are fired at the same barrier with the same speed as the protons. Will the width of the central maximum region (for the electrons) be the same as, larger than or smaller than that for the protons? Explain your answer.



The minima occur when

$a \sin\theta = \lambda$ +1 $\quad a =$ slit width $\quad$ and $\lambda = h/p$ +1

$\sin\theta = a\, h/p$

$= \frac{a h}{m v}$

smaller m for electrons +1 $\Rightarrow$ larger $\sin\theta$

$\Rightarrow$ larger θ +1

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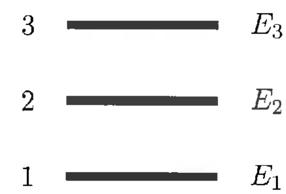
Question 6

An artificial atom has three energy levels as illustrated. The energies are:

$$E_1 = -12\,\text{eV}$$
$$E_2 = -8\,\text{eV}$$
$$E_3 = -5\,\text{eV}$$



a) Determine the smallest wavelength of light that can be emitted by the atom.

$E_{ph} = \frac{hc}{\lambda} = |\Delta E_{atom}|$ +1 +1

Need largest ΔE_{atom}

$1 \to 3$ $\quad \Delta E_{atom} = 7\,eV$

$= 7\,eV \times 1.6 \times 10^{-19}\,J/eV$

$= 1.12 \times 10^{-18}\,J$

+1

$\lambda = \frac{hc}{|\Delta E_{atom}|}$

$= \frac{6.63 \times 10^{-34}\,Js \times 3.0 \times 10^{8}}{1.12 \times 10^{-18}\,J}$

$\lambda = 177\,nm$

+2

b) Two photons are incident on the atom. Photon A has energy 4 eV and photon B has energy 6 eV. The atom is initially in the lowest energy state before the arrival of the photon. Which of the following (choose one) is true?

i) Photon A is more likely to be absorbed by the atom than photon B.

ii) Photon A is less likely to be absorbed by the atom than photon B.

iii) Photon A is equally likely to be absorbed by the atom as photon B.

Briefly explain your answer.

~~+3~~ +2 There is a 4eV transition ($1 \to 2$) $\Rightarrow$ 4eV can be absorbed

There is no 6eV " $\Rightarrow$ 6eV cannot be absorbed

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Question 7

The Bohr model predicts that the hydrogen atom energy levels are

$$E_n = -13.6\,\text{eV}\,\frac{1}{n^2}.$$

Consider the light *emitted* when the atom undergoes a transition from *any* state for which $n > 3$ to the $n = 3$ state.

a) Determine the largest possible wavelength for any transition to the $n = 3$ level.

Smallest gap => smallest energy => largest λ => $4 \to 3$

$E_{ph} = |\Delta E_{atom}|$

+1 [$\Delta E_{atom} = -13.6\,\text{eV}\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = -0.66\,\text{eV} \times 1.6\times10^{-19}\,\text{J/eV}$

$= -1.06\times10^{-19}\,\text{J}$] +3

+1 $E_{ph} = \frac{hc}{\lambda} = 1.06\times10^{-19}\,\text{J}$

$$\lambda = \frac{6.63\times10^{-34}\,\text{J·s} \times 3.0\times10^{8}\,\text{m/s}}{1.06\times10^{-19}\,\text{J}} = 1878\,\text{nm}$$

b) Describe which state will produce the smallest possible wavelength for any transition to the $n = 3$ level. Determine the energy of this state.

$n = \infty \to n = 3$

+1

energy $E_\infty = -13.6\,\text{eV}\,\frac{1}{\infty^2} = 0\,\text{eV}$

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