

Electromagnetic Theory II: Homework 10

Due: 12 March 2021

1 Reflection and transmission of waves

Consider waves traveling in one dimension along a medium with a single discontinuity; to the left the wave speed is v_1 and to the right v_2 . Consider a wave that is incident from the left.

- Express the amplitudes of the reflected and transmitted waves in terms of the amplitude of the incident waves and the velocities of the waves (rather than wavenumbers).
- Suppose that the wave speed to the left is 100 m/s and to the right 120 m/s. Determine the ratio of the amplitude of the reflected wave to the incident wave. Repeat this for the transmitted wave.
- Suppose that the wave speed to the left is 100 m/s and to the right 80 m/s. Determine the ratio of the amplitude of the reflected wave to the incident wave. Repeat this for the transmitted wave.
- Determine the ratio of the wave speeds so that the amplitude of the reflected wave is half that of the incident wave.

2 Energy flow and reflection and transmission

Two strings meet at a point. To the left of this point wavespeed is v_1 and to the right it is v_2 . Suppose that a wave with angular frequency ω is incident from the left.

- Show that the rate at which energy is reflected is always less than the rate at which energy is incident.
- Show that the rate at which energy is transmitted is always less than the rate at which energy is incident.
- Show that the rate at which energy is incident on the junction between the strings equals the rate at which it leaves the junction.

3 Polarization

Consider complex representations of vector waves,

$$\tilde{\mathbf{f}} = \tilde{\mathbf{A}}e^{i(kz - \omega t)}$$

where

$$\tilde{\mathbf{A}} = \tilde{A}_x \hat{\mathbf{x}} + \tilde{A}_y \hat{\mathbf{y}}$$

and $\tilde{A}_x = A_x e^{i\delta_x}$, $\tilde{A}_y = A_y e^{i\delta_y}$ with A_x, A_y and δ_x, δ_y all real.

- a) Suppose that $\tilde{A}_x = Ae^{i\pi/2}$ and $\tilde{A}_y = A$. Determine a real expression for the wave and describe how the vector evolves at $z = 0$.
- b) Suppose that $\tilde{A}_x = A$ and $\tilde{A}_y = Ae^{i\pi/2}$. Determine a real expression for the wave and describe how the vector evolves at $z = 0$.
- c) Suppose that $\tilde{A}_x = Ae^{i\delta}$ and $\tilde{A}_y = Be^{i\delta}$ where the two phases are identical. Describe the polarization of this wave.