Phys 111L
Fall 2021

Laboratory 4: Vectors – Prelab

1 Adding and subtracting vectors

Consider the two vectors

$$\vec{\mathbf{A}} = 30.0\,\mathrm{m} \text{ along East and}$$
$$\vec{\mathbf{B}} = 20.0\,\mathrm{m} \text{ at an angle of } 60^\circ \text{ North of West.}$$

a) Decompose $\vec{\mathbf{A}}$ into x and y components.

b) Decompose $\vec{\mathbf{B}}$ into x and y components.

c) Determine the magnitude of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.

d) Determine the direction (with respect to the $+x$ axis) of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.

Laboratory 4: Vectors – Experiment/Tutorial

Vectors and vector algebra are used throughout classical physics. This laboratory exercise will illustrate vector algebra using physical and graphical methods and compare the results to those that can be obtained with purely mathematical constructions.

1 Walking displacement vectors

Displacement vectors describe the location of an object at a later instant relative to its location at an earlier instant. These vectors provide a useful conceptual illustration of vector addition: two displacements can be combined in succession to provide an overall displacement in the same way that two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, can be added, giving the sum $\vec{\mathbf{A}} + \vec{\mathbf{B}}$. This exercise illustrates such vector addition via walking various displacements.

Consider the following sequence of displacements:

$$\vec{\mathbf{A}} = 5.0\,\mathrm{m} \text{ due East and}$$
$$\vec{\mathbf{B}} = 3.2\,\mathrm{m} \text{ at an angle of } 20^\circ \text{ North of West}$$

and let $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. This is illustrated approximately in Fig. 1.

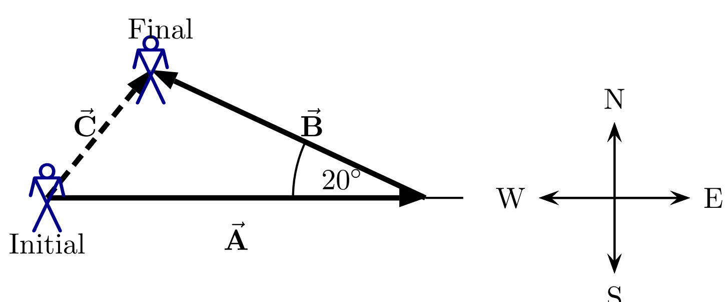

Figure 1: Successive displacements

The aim of this exercise will be to follow (by walking) the two displacements successively and to measure the resulting overall displacement vector $\vec{\mathbf{C}}$. The measured overall displacement will then be compared to a calculated overall displacement.

a) Take some rope, a meter stick and a protractor outside. Stretch the rope in a straight line approximately East/West. This will establish what we regard as "East."

b) Stand at one end of the rope and leave a "marker" object at this location. This is the **initial** location.

c) Follow the displacement vector $\vec{\mathbf{A}}$ by walking 5.0 m along the rope. At the end of this you have reached an intermediate location.

d) Starting at this intermediate location, follow the displacement described by $\vec{\mathbf{B}}$. This takes you to a **final** location. Mark this location.

e) Determine the overall displacement from your initial to final locations. This should consist of **both** a **distance** and a **direction** (e.g. angle measured from "East" toward "North" using the "line of sight" from the initial to the final location). The "line of sight" is illustrated for a different example in Fig. 2.

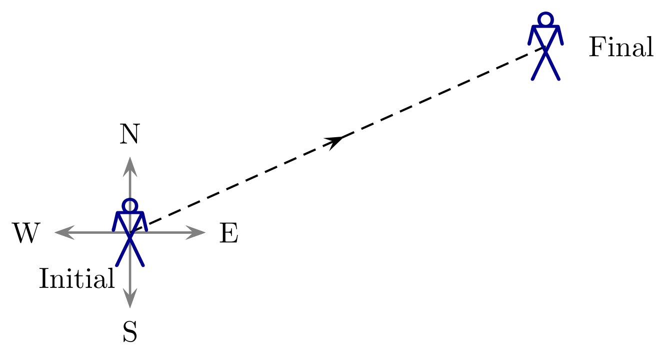

Figure 2: Overall displacement

Record the data for the displacement below.

The rest of this exercise should be conducted in the lab room.

f) You will now use components to calculate the magnitude and direction of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ **without using the results from the walking activity.** Let "East" correspond to the x axis and "North" the y axis when you do this. Determine the components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ and use them to find the components of $\vec{\mathbf{C}}$. Then use these to sketch the vector $\vec{\mathbf{C}}$ and obtain its magnitude and angle. The table beneath might be useful.

Vector	x component	y component
$\vec{\mathbf{A}}$		
$\vec{\mathbf{B}}$		
$\vec{\mathbf{C}}$		

g) Do the magnitude and direction of the calculated $\vec{\mathbf{C}}$ agree with those of the measured displacement as determined via walking?

h) Using the notation that A is the magnitude of $\vec{\mathbf{A}}$, B is the magnitude of $\vec{\mathbf{B}}$, etc, $\ldots$, is it true that $C = A + B$ in this exercise? Explain your answer.

i) Is it true that $C^2 = A^2 + B^2$ in this exercise? Explain your answer.

2 Vector addition: graphical methods versus algebraic methods

A more accurate comparison of graphical methods and algebraic methods of adding vectors can be illustrated by drawing and adding vectors accurately on a sheet of paper. Consider the three vectors:

$$\vec{\mathbf{A}} = 8.0\,\text{cm along the } x \text{ axis,}$$

$$\vec{\mathbf{B}} = 6.0\,\text{cm at an angle of } 45^\circ \text{ counterclockwise from the } x \text{ axis and}$$

$$\vec{\mathbf{C}} = 12.0\,\text{cm at an angle of } 120^\circ \text{ counterclockwise from the } x \text{ axis.}$$

Let $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$. The aim of this exercise is to determine $\vec{\mathbf{D}}$ graphically and algebraically

a) A graphical method for determining $\vec{\mathbf{D}}$ would first add $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}$ using the "head-to-tail" method. Draw these three vectors "head-to-tail" as accurately as possible (using a ruler and a protractor) on a separate sheet. Use this to draw $\vec{\mathbf{D}}$.

b) Determine the magnitude of $\vec{\mathbf{D}}$ graphically (i.e. by measuring from your drawing). Record this below.

c) Draw the x and y components of $\vec{\mathbf{D}}$ on your diagram. Write the results below.

d) Without using your drawing, calculate the components and magnitude of $\vec{\mathbf{D}}$ mathematically from the components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ using algebra and trigonometry (this is the "algebraic" method). How do these compare to result that you obtained graphically?

3 Vector subtraction: graphical methods versus algebraic methods

Vector subtraction is a variant of vector addition. Consider any two vectors $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ and define the difference between these as

$$\Delta\vec{\mathbf{v}} := \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 \tag{1}$$

One way of constructing the difference is involves addition of a negative vector. Thus

$$\begin{aligned} \Delta\vec{\mathbf{v}} &= \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 \\ &= \vec{\mathbf{v}}_2 + (-\vec{\mathbf{v}}_1) \end{aligned} \tag{2}$$

Thus one can perform the subtraction by adding the vectors $\vec{\mathbf{v}}_2$ and $-\vec{\mathbf{v}}_1$. Alternatively, the difference between the vectors must satisfy

$$\vec{\mathbf{v}}_2 = \vec{\mathbf{v}}_1 + \Delta\vec{\mathbf{v}} \tag{3}$$

and, given the two vectors $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$, one can try to find $\Delta\vec{\mathbf{v}}$ that satisfies Eq. (3).

For this exercise consider the vectors

$\vec{\mathbf{v}}_1 = 5.0\,\text{cm}$ at an angle of 30° counterclockwise from the x axis and

$\vec{\mathbf{v}}_2 = 5.0\,\text{cm}$ at an angle of 150° counterclockwise from the x axis.

a) On a separate sheet of paper, draw a set of x and y axes and draw $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ as accurately as possible.

b) The vectors $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ have the same magnitude. Does this mean that $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = 0$? Explain your answer.

c) Use the scheme of Eq (2) to draw $\Delta\vec{\mathbf{v}}$ as accurately as possible on the same sheet of paper. Use your drawing to describe the direction of $\Delta\vec{\mathbf{v}}$ and to determine its magnitude. Record these below.

d) Calculate the components of $\Delta\vec{\mathbf{v}}$ using trigonometry and algebra. Use the result to describe the magnitude and direction of $\Delta\vec{\mathbf{v}}$. How do these calculated results compare to your measured results?

e) Use the "head-to-tail" method of addition and accurate drawings of $\vec{\mathbf{v}}_1$ and $\Delta\vec{\mathbf{v}}$ to construct $\vec{\mathbf{v}}_1 + \Delta\vec{\mathbf{v}}$ graphically. How does the resulting vector compare to $\vec{\mathbf{v}}_2$? What does this imply (correct, incorrect, maybe correct) about Eq. (3)?