

Fri. *Paper due by midnight.

* Turn in previous first draft.

Fundamental changes brought by atomic clocks

One of the motivations at the time for developing atomic clocks was the definition of the unit of time: the second.

Prior to the 1960s the definition had been related to the rotational motion of Earth, as observed via the apparent motion of Sun, Moon, Stars,...

The definition came to be

$$1 \text{ second} = \frac{1}{86400} \text{ of the mean solar day}$$

Recall that the solar day refers to the period from one occurrence of noon to the next. The mean solar day takes apparent seasonal variations into account via the equation of time and other corrections.

By the mid 20th century precision measurements revealed that the rate of rotation of Earth varies during the seasons. This led to a redefinition of the second as a particular fraction of the tropical year of 1900 - this redefinition appeared in the 1950s.

1 Reference standard for time

Throughout this course we have encountered various periods of timekeeping:

Ancient times: Sundials, water clocks,

Medieval times: Sundials, water clocks, verge-and-foliot clocks

17th – 18th centuries: Pendulum and balance-spring clocks, precision chronometers

19th century: Time standardization, electrical time-signaling

Early 20th century: Quartz timing standards and clocks

Mid 20th century – present: Atomic clocks

- During each period what served as the ultimate authority or reference against which the accuracy of any device could be checked? What are the issues with this reference?
- During each of these periods could one use an example of an actual clock itself as a reference? What might the issues with this be?
- During each period were there any better alternative ultimate references?

Answer: a) During all but the last period, the Sun or stars were the reference against which any timekeeping device should be checked.

Issues: * difficulties with actual observations

* limited precision of actual observations

* variability of Earth's rotation - numerous corrections

During the last * - Cesium or Rubidium atoms

b) This would require a single specially designated example of such a clock. The issues would be:

* access to this to compare other devices

* if the example breaks down then?

* any copy would contain imperfections

c) Not that were known

Once atomic clocks were developed it became clear that the constituent atoms could serve as better references. The frequency measured with the particular transition of the cesium atom was (1957)

$$9\,192\,631\,770 \pm 20 \text{ Hz}$$

Then the second was redefined as (1967/8):

1 second = 9 192 631 770 cycles of this particular cesium energy transition.

Time and physical locations

One can use timing of electromagnetic signals to determine locations. The basic situation and strategy is:



transmitter at
known location



receiver at
unknown location

- 1) a transmitter at a known location emits an electromagnetic wave (e.g. radio signal) at a known time.
- 2) a receiver whose clock is synchronized with that of the transmitter detects the signal at a later time.
- 3) the time taken for the signal to travel is determined.
- 4) then

$$\text{speed} = \frac{\text{distance}}{\text{travel time}} \Rightarrow \text{distance} = \text{speed} \times \text{travel time}$$

$$\Rightarrow \text{distance} = 3.0 \times 10^8 \text{ m/s} \times \text{travel time}$$

allows for calculation of the distance from the transmitter to the receiver.

2 Location finding by time measurements

A transmitter sends a radio wave signal at exactly 1:00pm and 0.0000000 s. The signal arrives at a detector at 1:00pm and 0.0000200 s.

- Determine the distance from the transmitter to the receiver.
- Suppose that the timing devices/clocks were only accurate up to one millionth of a second. This means that the true time at which the signal could have arrived is between 1:00pm and 0.0000190 s and 1:00pm and 0.0000210 s. For each of the extremes determine the calculated distance between the transmitter and the receiver. What would the inaccuracies in position be?

Answer: a)
$$\text{distance} = 3.0 \times 10^8 \text{ m/s} \times \frac{0.0000200 \text{ s}}{2 \times 10^{-5} \text{ s}}$$
$$= 6000 \text{ m} = 6 \text{ km.}$$

b) for the shorter time

$$\begin{aligned} \text{distance} &= 3.0 \times 10^8 \text{ m/s} \times 1.9 \times 10^{-5} \text{ s} \\ &= 5700 \text{ m} \end{aligned}$$

for the longer time

$$\begin{aligned} \text{distance} &= 3.0 \times 10^8 \text{ m/s} \times 2.1 \times 10^{-5} \text{ s} \\ &= 6300 \text{ m} \end{aligned}$$

Thus we know the position with an accuracy of $\pm 300 \text{ m}$ if the timing error is 0.0000010 s. ■

We see that very small (by usual standards) inaccuracies in timing can lead to large inaccuracies in position. Similar calculations reveal

| Timing accurate up to | Distance known accurately up to |
|--|---|
| 1.0s | $3.0 \times 10^8 \text{ m} = 300,000 \text{ km} = 188,000 \text{ mi}$ |
| 0.1s | $3.0 \times 10^7 \text{ m} = 30,000 \text{ km} = 18,800 \text{ mi}$ |
| 1.0ms = 0.001 s | $3.0 \times 10^5 \text{ m} = 300 \text{ km} = 188 \text{ mi}$ |
| 1.0 $\mu\text{s} = 0.000001 \text{ s}$ | 300 m = 1000 ft |
| 10ns = 10^{-8} s | 3 m = 10 ft |
| 1.0ns = 10^{-9} s | 0.30 m \approx 1 ft. |
| 0.1ns = 10^{-10} s | 0.03 m \approx 1 in |

Thus in order to use such schemes to measure positions within, say, an accuracy of 10ft requires extremely precise timing. The various clocks involved can only afford to be off by miniscule fractions of a second.