Essl 290 2021

Lecture 25

Fri: No Class

Weds 27 Oct - Res Paper 2 Draft 1

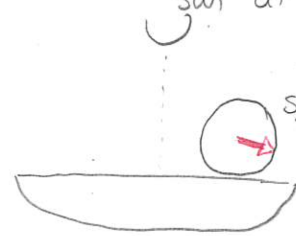
Weds 3 Nov - Res Paper 2 Draft 2

Longitude, navigation and time

One method for finding longitude differences between two locations involves synchronized clocks. The typical situation involves a ship in the open ocean that needs to know the longitude difference between the ship and a port. The process is:

1) ship synchronizes a clock with a clock in the port. This clock must run accurately with no interruption. This clock will keep a record of the exact time in the port

2) the ship observes local noon at its location at the same instant the ship reads the time on the synchronized clock.



3) this gives the local time in port and thus the amount of time between local noon in the port and local noon at the ship.

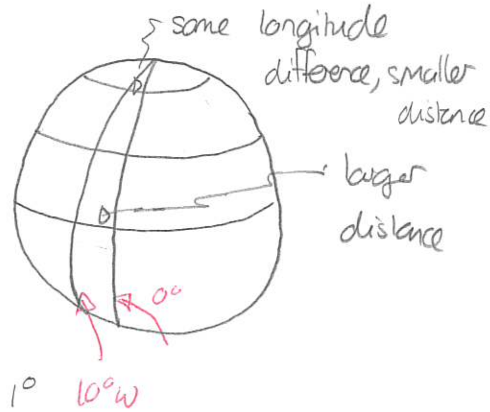
For example if the synchronized clock reads 10:40am then we know that local noon on the ship has occured 1hr 20min before local noon on the ship. This corresponds to a rotational difference of 1hr 20min beneath the Sun between the locations. This is $1 \times 15^\circ + \frac{1}{3} \times 15^\circ = 20^\circ$. So the ship is 20° east of the port.

The crucial part to this is the necessary accuracy of the clock. We know that

$$1^\circ \leadsto 4\,\text{min} \quad \Rightarrow \quad 1\,\text{min} \sim 0.25^\circ \text{ longitude difference.}$$

In terms of actual distance this depends on the latitude

* further from the equator 1° difference is smaller in distance
* closer to " " 1° " " larger in distance



Geometry and trigonometry give the result that at latitude θ, the circumference of the entire line of latitude is $2\pi R\cos\theta$ where R is the radius of Earth.

Then this is divided into 360 intervals of 1° longitude. So two lines of longitude separated by 1° are distance

$$\frac{2\pi R\cos\theta}{360^\circ}$$

apart. At the equator Earth's radius is 6378 km (3963mi). So

$$1^\circ \text{ longitude difference} = \frac{2\times\pi\times 3963\text{mi}\times\cos\theta}{360^\circ} = 69.17\text{mi}\cos\theta$$

A table gives:

latitude	1° longitude difference (4min time)
0° (equator)	69.17mi
39° (Grand Jct)	53.76mi
51.5° (London)	43.05mi.

Inverting gives the time difference per mile of longitude difference.

latitude	time difference for 1mile longitude diff
0°	$0.058\,min = 3.5s$
39°	$0.074\,min = 4.5s$
51.5°	$0.093\,min = 5.6s$

So if we want to know to know E-W distance precisely at the level of one mile then the timing must be accurate to a few seconds.

1 Navigation timekeeping accuracy

The Longitude Act (Great Britain, 1714) required a clock that would lose at most 2 minutes on a voyage from England to the West Indies and back. The total time that the voyage might take could be anywhere between eight and ten weeks.

a) Suppose that the voyage took eight weeks. How many seconds could the clock lose per day?

The clock can lose $2 \times 60s = 120s$ in that time. Then the number of days is $7 \times 8 = 56$ days. So it can lose

$$\frac{120s}{56\text{days}} = 2.1s \text{ per day.}$$

b) Suppose that the voyage took ten weeks. How many seconds could the clock lose per day?

Similar calculation. There are 70 days. So the clock can lose

$$\frac{120s}{70\text{days}} = 1.7s \text{ per day.}$$

c) Would a typical pendulum clock of 1700 be able to accomplish this accuracy?

No, it lost 15s per day.

d) Consider typical daily tasks that require you to look at ~~your watch~~ a clock. Would a typical pendulum clock be accurate enough? Would there be any need for a clock with the accuracy required by the Longitude Act?

In general one only needs to know the time to within a few minutes. If one was prepared to check the watch once per week, a pendulum clock would be adequate.

Thus we need a clock that is roughly ten times as accurate as the pendulum clocks or the balance spring clocks developed in the late 1600s. In this case the increased accuracy is necessary for a technical task - finding longitude.

Navigation demands

Why was there a demand for such accurate clocks? The demand came from Europe. By the end of the 1400s Europeans had begun to explore beyond the coasts of Europe, the Mediterranean and the Black Sea.

Demo: Wikipedia Age of Discovery
Show map of Maritime Traffic in Mediterranean.

Navigation in these cases could be done using
1) landmarks
2) established routes across open water.

By the late 1400s Portugal started to explore the Atlantic and the coast of Africa. Initially much of the travel was
1) roughly North-South
2) often close to the coast.

Q: What ~~navigational~~ nautical trips require significant E-W travel?
~~What might have motivated this?~~

By the early 1500s Europeans had pioneered naval routes to the Americas, across the Indian Ocean and Pacific Oceans. ~~These required~~ significant traverses across open water, spanning multiple degrees of longitude:

Portugal → N. America ~60° E-W
Portugal → Brazil ~30° E-W
Kenya → India ~30° E-W

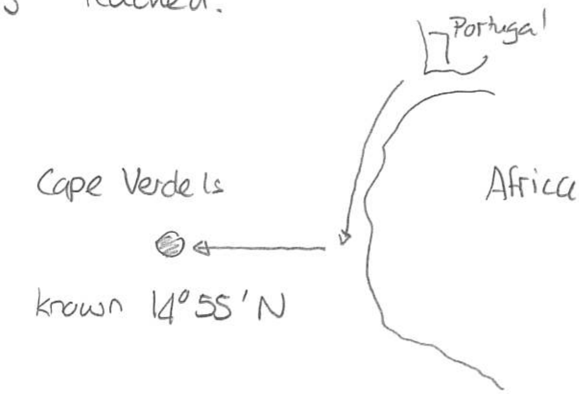
S. America → Phillipines 160° E-W

Knowledge of longitude on such trips would clearly be important. Without any clocks or celestial navigation two ways to navigate E-W would be:

1) dead-reckoning requires known:

* starting point
* direction
* speed.

2) sailing the latitude - sail along a known coast until desired latitude is reached. Then sail along the latitude.



Both clearly present challenges and significant risks without any knowledge of longitude.