Lecture 23

→ HW

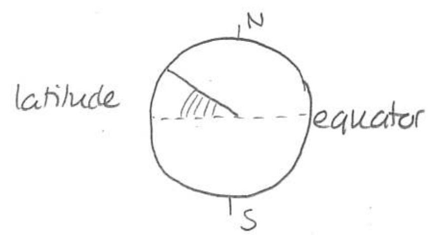
Mon: Mondschein 110 – 117

Barnett. 104 – 111

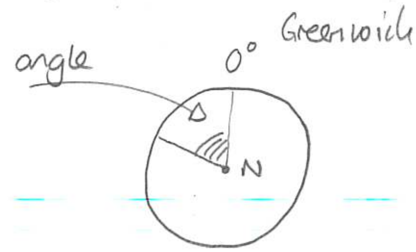
Determining locations

Locations on Earth are specified in terms to two angular numbers

1) Latitude. - angle above or below equator. This runs from 90°S to 90°N

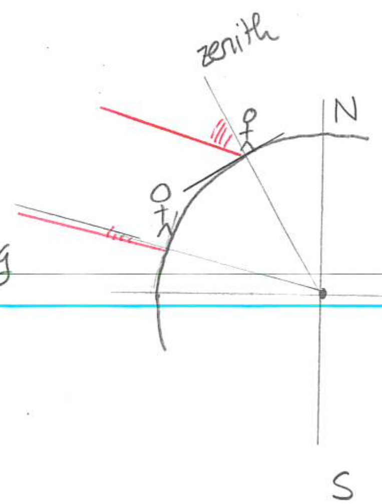


2) Longitude - angle about Earths axis This runs from 180° E to 180° W with 0° at Greenwich



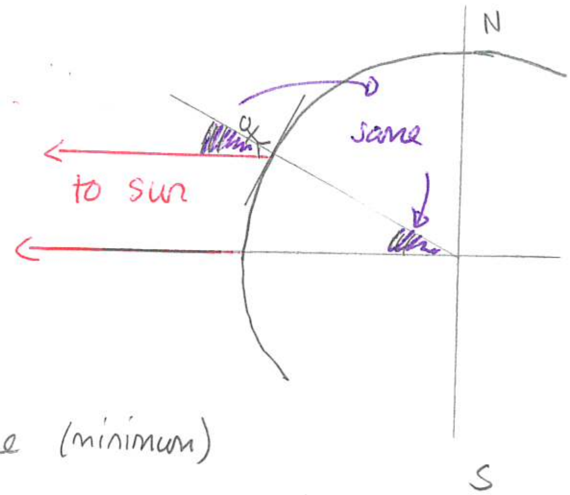
Determing latitude

Latitude can be determined by observing the apparent position of the Sun at local noon. This angle will clearly vary depending on the latitude of the observer. The diagram illustrates this for two observers along the same line of longitude.



To see how one can determine latitude in this way consider observation at the equinox, where Sun is directly above the equator

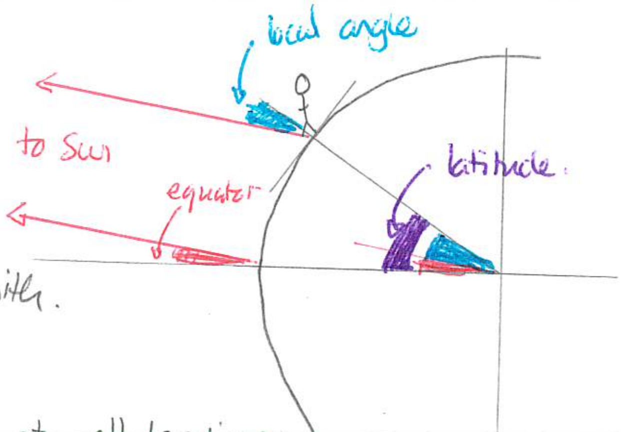
Then geometry shows that, on this day, the angle between Sun and the zenith at local noon is the same as the latitude angle.



The actual observation may involve the (minimum) angle between Sun and horizon plus some geometrical calculations.

On days that are not the equinox, we need to know the angle between Sun and zenith at the equator. This can be calculated knowing the day of the year. Then geometry shows that in the northern hemisphere beyond the tropic

latitude = local angle from zenith + equator angle from zenith.



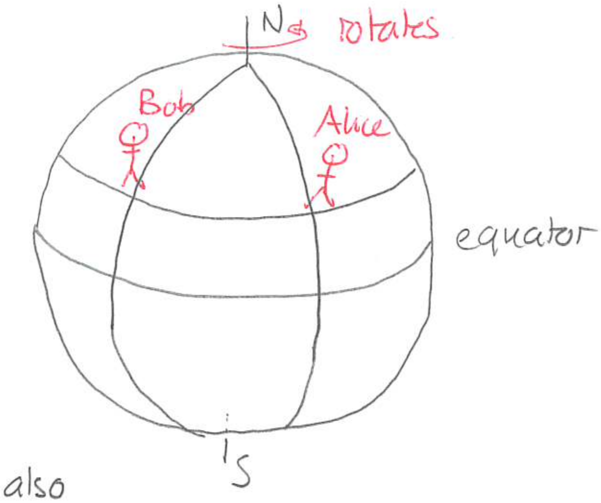
Similar calculations are available at all locations.

In general:

Latitude can be determined from observations of Sun's position relative to the local zenith at local noon. This requires knowing the day of the year and some simple geometry

Determining longitude

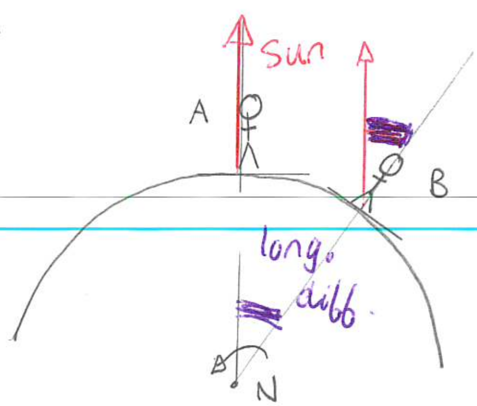
We cannot determine longitude by a single simple observation of Sun's position. Consider two observers along the same line of latitude but at different longitudes. Alice is east of Bob. During any particular day the apparent motion of Sun will appear exactly the same for Alice as it does for Bob.



So whatever position Alice observes will also be observed by Bob at some point during that day.

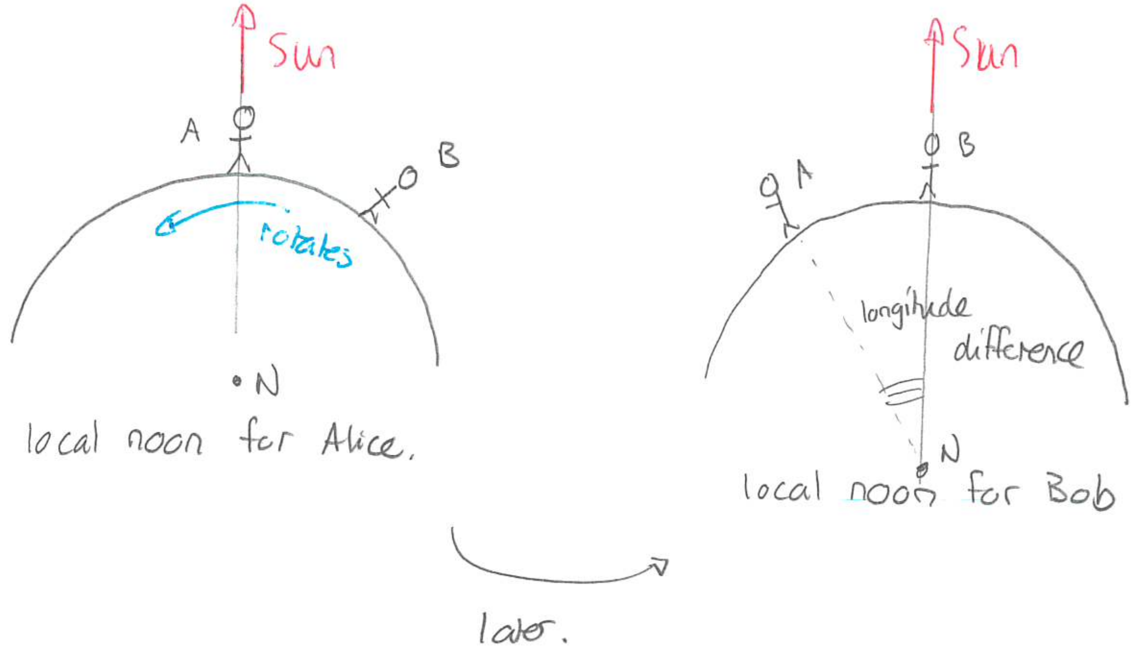
However, there is one crucial difference. That is the times at which those positions will be observed will differ for the two observers.

Suppose that Alice observes the Sun at her local noon (when it is highest) and that Bob observes the Sun at the same instant. To simplify the geometry assume that the Sun passes directly overhead on that day.



At this instant the angle between the local zenith and Sun is different. If we could arrange for them to observe ~~at the same instant then we could~~ use the angular measurements to get the longitude difference

In practice the process is different. We observe the Sun a local noon for each observer. This will work at all latitudes and all days.



If we can learn how much time passes between the two observations of local noon then we can determine the longitude difference easily.

1 Longitude and local noon observations

This exercise will consider two people, Alice and Bob, at different locations. Each is equipped to measure the angle of Sun with respect to the local zenith (the vertical direction at their location). They each determine the instant when local noon occurs. At a particular location, local noon is the moment when Sun is highest in the sky (or when the angle between Sun and the zenith is smallest). They can determine the time difference between the occurrences of these local noons very precisely.

a) If Alice and Bob are along the same line of longitude but at different latitudes, will there be any difference in the times at which they observe local noon? Explain your answer.

No, Sun is at its highest at all points on the same line of longitude at the same time.

b) If Alice and Bob are along the same line of latitude but at different longitudes, will there be any difference in the times at which they observe local noon? Explain your answer.

Yes, this was illustrated earlier.

In general there will be a time difference between the moments when local noon occurs. To translate this into a difference in longitude, consider Earth's rotational motion.

c) By how many degrees does the Earth rotate in one day?

$360°$

d) By how many degrees does Earth rotate in one hour?

$$\frac{360°}{24\,\text{hr}} = 15° \text{ per hour}$$

e) How many minutes does it take for Earth to rotate through 1°?

~~There are 60 minutes in one hour. Earth rotates through~~ 15° in this time. So it takes

$$\frac{60\,\text{min}}{15°} = 4\,\text{min per degree.}$$

Earth rotates through 4 min per degree

f) Suppose that local noon occurs three hours earlier for Alice than it does for Bob. What is the difference in their longitudes?

15° per hour $\Rightarrow$ $3 \times 15^\circ = 45^\circ$

for three hours.

g) Suppose that local noon occurs five hours and 20 minutes earlier for Alice than it does for Bob. What is the difference in their longitudes?

This is 5 hrs and $\frac{1}{3}$ hr

$5 \times 15^\circ = 75^\circ$

$\frac{1}{3} \times 15^\circ = 5^\circ$

add $75^\circ + 5^\circ = 80^\circ$

h) Suppose that local noon occurs seven hours and 12 minutes earlier for Alice than it does for Bob. What is the difference in their longitudes?

Convert to minutes $7 \times 60\text{min} + 12\text{min} = 432\text{min}$.

We know $4\text{min} \sim 1^\circ$ so. number of degrees is

$$\frac{432\text{ min}}{4\text{ min}/^\circ} = 108^\circ$$

i) Plymouth, England is at longitude 4.1427° W and Bridgetown, Barbados (in the Caribbean) is at 59.6132° W. Determine the difference in time between the local noons at these two locations.

The difference in longitudes is $59.6132^\circ - 4.1427^\circ = 55.4705^\circ$

Then the time difference is $55.4705^\circ \times 4\text{min} = 222\text{ min}$

$= 3\text{hr } 42\text{min}.$

We see that we can determine longitude differences via:

* Observe sun at local noon
  - time difference of 4 min ⇒ 1° longitude difference
  - time difference of 1 hr ⇒ 15° longitude difference

So, to determine the difference in longitude between two locations, we need only determine the time difference between local noons at those locations.