Statistical and Thermal Physics: Homework 5

Due: 7 February 2020

1 Differentiation of multivariable functions

Consider an ideal gas and suppose that a function of interest satisfies

$$F = PV^n$$

where n is an integer.

a) Determine $\left(\frac{\partial F}{\partial T}\right)_V$ and $\left(\frac{\partial F}{\partial T}\right)_P$. Eliminate T to rewrite the results in terms of P and V.

b) Determine values of n for which $\left(\frac{\partial F}{\partial T}\right)_V = \left(\frac{\partial F}{\partial T}\right)_P$.

2 Differentiation identities

Consider three variables x, y, z that are not independent. This means that they are related by some function, which could be written $z = z(x, y)$ or $y = y(x, z)$ or $x = x(y, z)$ depending on the choice of independent variables. These functions can all be differentiated with respect to their variables; the derivatives must be related. This exercise will result in general relationships between these derivatives that are always satisfied.

a) To illustrated this let $z = x^2 y$. So $z(x, y) = x^2 y$. Find expressions for $x = x(y, z)$ and $y = y(x, z)$. Determine expressions for

$$\left(\frac{\partial z}{\partial y}\right)_x, \quad \left(\frac{\partial z}{\partial x}\right)_y, \quad \left(\frac{\partial x}{\partial y}\right)_z, \quad \left(\frac{\partial x}{\partial z}\right)_y, \quad \left(\frac{\partial y}{\partial x}\right)_z, \text{and} \quad \left(\frac{\partial y}{\partial z}\right)_x.$$

According to your results how are $\left(\frac{\partial z}{\partial y}\right)_x$ and $\left(\frac{\partial y}{\partial z}\right)_x$ related to each other? How about $\left(\frac{\partial z}{\partial x}\right)_y$ and $\left(\frac{\partial x}{\partial z}\right)_y$?

b) To prove these relationships *for any function*, first consider x and y as the independent variables. Express dz in terms of dx and dy, using appropriate partial derivatives. Note that this must be done for any function $z(x, y)$, not just the special case above. Then express dx in terms of dy and dz, using appropriate partial derivatives. Substitute this into the general expression for dz; this will give an expression for dz in terms of dx and dz. Use the expression to show that

$$\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y = 1 \quad \text{and} \tag{1}$$

$$\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial z}{\partial y}\right)_x. \tag{2}$$

These identities will be important throughout the subject.

c) Check that the identities are valid for the function $z(x, y) = x^2 y$.

d) There is nothing special about the order of the variables in Eqs. (1) and (2). You could permute the variables as $x \to y, y \to z$ and $z \to x$ and the identities must still be valid. Do this and check the resulting identities for $z = x^2 y$.

e) Check these identities for an ideal gas, with $z \to P, x \to T$ and $y \to V$.

3 Heat capacities for an ideal gas

Consider monoatomic gases. For an ideal gas,

$$E = \frac{3}{2} NkT.$$

a) Using the energy, E, determine c_V and c_P for this gas.

b) Explain, in terms of the energy involved in the processes to measure the two heat capacities, why you would expect $c_P > c_V$.