Statistical and Thermal Physics: Homework 3
Due: 31 January 2020

1 Thermal expansion
The linear thermal expansion coefficient describes the increase in length of a homogenous material as its temperature changes and is defined by

$$\alpha_1 := \frac{1}{L} \frac{\partial L}{\partial T}$$

where $L$ is the length of the material. The subscript in $\alpha_1$ is not standard notation; it is included to distinguish the linear thermal expansion coefficient from the volume thermal expansion coefficient.

a) For copper, $\alpha_1 = 1.65 \times 10^{-5} \text{ K}^{-1}$. Determine the amount by which a copper rod of length $10 \text{ m}$ will expand if it is heated from $-10^\circ \text{C}$ to $30^\circ \text{C}$, assuming that $\alpha$ is independent of temperature.

b) A rectangular sheet of any material will expand as its temperature increases. Here the coefficient of area expansion is defined by

$$\gamma := \frac{1}{A} \frac{\partial A}{\partial T}$$

where $A$ is the area of the material. Show that

$$\gamma = 2\alpha_1.$$ 

Determine the amount by which the area of a rectangular copper roof, whose sides are $10 \text{ m}$ and $4 \text{ m}$ increases if it is heated from $-10^\circ \text{C}$ to $30^\circ \text{C}$.

2 Thermal expansion coefficient for a van der Waals gas
Determine an expression for the thermal expansion coefficient for a van der Waals gas. Hint: note that differentiation of $V$ w.r.t. $T$ will be difficult. There is an identity that we encountered in class that will make this easier.

3 Isothermal compressibility of an ideal gas
Determine an expression for the isothermal compressibility of an ideal gas, in terms of $N, P$ and $T$. Show that it is positive.
4 Equation of state for a solid

The state of a solid material can be described by the same variables as for a gas. Suppose that the equation of state of the solid is

\[ V = V_0(1 + aT - bP) \]

where \( V_0 \) is a constant equal to the volume when pressure and temperature are zero and \( a \) and \( b \) are constants that are very small.

a) Determine expressions for the isothermal compressibility and the isobaric expansion coefficient.

b) Suppose that \( a \) and \( b \) are so small that at typical temperatures \( aT \ll 1 \) and \( bP \ll 1 \). Determine approximate expressions for the isothermal compressibility and the isobaric thermal expansion coefficient in this case. Are they approximately constant or not?