Phys 362
Spring 2020

Statistical and Thermal Physics: Class Exam I

5 March 2020

Name: ______________________ **Total:** /50

Instructions

- There are 6 questions on 8 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$R = 8.31\,\text{J/mol K}$ $\quad N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$ $\quad k = 1.38 \times 10^{-23}\,\text{J/K}$ $\quad 1\,\text{atm} = 1.01 \times 10^5\,\text{Pa}$

Question 1

The atmosphere of Titan consists nearly entirely of molecular nitrogen (mass of 1 mol = 0.028 kg). The surface pressure is about 147×10^3 Pa and the surface temperature is about 94 K. Determine the density of Titan's atmosphere at its surface.

We can get the number of particles per unit volume, the number of moles per unit volume and then the mass per unit volume: Then

$PV = NkT \Rightarrow \frac{N}{V} = \frac{P}{kT}$ number per volume

$\Rightarrow$ number mols per volume $= \frac{1}{N_A}\frac{N}{V} = \frac{P}{N_A kT}$

$\Rightarrow$ mass per volume $= m_{mol} \frac{P}{N_A kT}$

$$= \frac{0.028\,\text{kg mol}^{-1} \times 147 \times 10^3\,\text{Pa}}{6.02 \times 10^{23}\,\text{mol}^{-1} \times 1.38 \times 10^{-23}\,\text{J/K} \times 94\,\text{K}}$$

$= 5.3\,\text{kg/m}^3$

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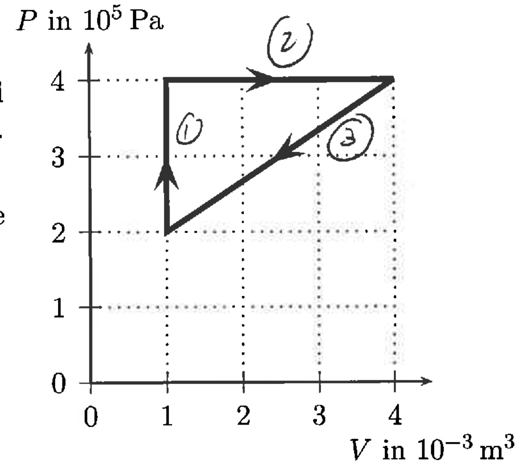
Question 2

A heat engine operates by having a monoatomic ideal gas undergo the process indicated on the PV diagram.



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a) Determine the work done by the engine in one cycle.

W = area under curve / inside cycle

$= \frac{1}{2} \times 3 \times 2$ blocks $\times$ $\underbrace{\text{work per block}}$ → $1 \times 10^5 \,\text{Pa} \times 1 \times 10^{-3} \text{m}^3$

$= 3 \times 100\,\text{J}$

$= 300\,\text{J}$

b) Determine the efficiency of the engine.

Need $\Delta E = Q + W$ for each leg.

$E = \frac{3}{2} NkT = \frac{3}{2} PV$

So $\Delta E = \frac{3}{2} \Delta (PV)$

Along ① $W_1 = 0$

$\Delta E_1 = \frac{3}{2}(P_f V_f - P_i V_i) = \frac{3}{2} V \Delta P = \frac{3}{2}\, 1 \times 10^{-3} \times 2 \times 10^5 \,\text{Pa}$

$= 300\,\text{J}$

$Q_1 = 300\,\text{J}$

Question 2 continued ...

Along 2

$W_2 = -P\Delta V$

$= -4\times10^5 Pa \times 3\times10^{-3} m^3$

$= -1200 J$

Leg	ΔE	Q	W
1	300J	300J	0J
2	1800J	3000J	-1200J
3	-2100J	-3000J	900J

$\Delta E_2 = \frac{3}{2}\Delta(PV) = \frac{3}{2}P\Delta V$

$= \frac{3}{2}\, 4\times10^5 Pa \times 3\times10^{-3} J$

$= 1800J$

Along 3 $W_3 =$ area under PV

$= (6+3)$ blocks $\times$ W per block

$= 900J$

$\Delta E_3 = \frac{3}{2}(P_f V_f - P_i V_i) = \frac{3}{2}(200J - 1600J)$

$= -2100J$

$\eta = \dfrac{W}{\text{heat supplied}} = \dfrac{300J}{3300J} = \dfrac{1}{11} = 0.091$

↳ stages 1,2

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Question 3

Answer either part a) or part b) for full credit for this problem.

a) The isothermal compressibility of a gas is given by

$$\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

and the thermal expansion coefficient is

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

Starting with an expression for $\mathrm{d}P$ in terms of $\mathrm{d}T$ and $\mathrm{d}V$, show that

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{\alpha}{\kappa}.$$

$$dP = \left(\frac{\partial P}{\partial T}\right)_V dT + \left(\frac{\partial P}{\partial V}\right)_T dV$$

Now

$$dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP$$

$$\Rightarrow \quad dP = \left(\frac{\partial P}{\partial T}\right)_V dT + \left(\frac{\partial P}{\partial V}\right)_T \left[\left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP\right]$$

$$= \underbrace{\left\{\left(\frac{\partial P}{\partial T}\right)_V + \left(\frac{\partial P}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_P\right\}}_{=0} dT + \{\ldots\} dP$$

$$\left(\frac{\partial P}{\partial T}\right)_V = -\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial V}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \Big/ \left(\frac{\partial V}{\partial P}\right)_T = -\frac{\alpha V}{-\kappa V} = \alpha/\kappa$$

Question 3 continued ...

b) Starting with $\mathrm{d}E = \delta Q - P\mathrm{d}V$ and the definition of enthalpy $H = E + PV$, show that

$$c_V = \left(\frac{\partial H}{\partial T}\right)_V - V\left(\frac{\partial P}{\partial T}\right)_V.$$

$\delta Q = dE + PdV$ $\qquad$ $E = H - PV$

$dE = dH - PdV - VdP$

$\Rightarrow \quad \delta Q = dH - \cancel{PdV} - VdP + \cancel{PdV}$

$= dH - VdP$

$= \left(\frac{\partial H}{\partial T}\right)_V dT + \left(\frac{\partial H}{\partial V}\right)_T dV - V\left[\left(\frac{\partial P}{\partial T}\right)_V dT + \left(\frac{\partial P}{\partial V}\right)_T dV\right]$

$= \underbrace{\left\{\left(\frac{\partial H}{\partial T}\right)_V - V\left(\frac{\partial P}{\partial T}\right)_V\right\}}_{C_V} dT + \{\ldots\} dV$

$\Rightarrow \quad C_V = \left(\frac{\partial H}{\partial T}\right)_V - V\left(\frac{\partial P}{\partial T}\right)_V$

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Question 4

The entropy for a particular gas is

$$S(E, V, N) = Nk \ln(E^{\beta}) + Nk \ln(V)$$

where β is a constant.

a) Determine an expression for the temperature of the gas and use it to determine a relationship between the energy and temperature of the gas.

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{V,N} = Nk \frac{1}{E^{\beta}} \beta E^{\beta-1} = \frac{\beta N k}{E}$$

$$\Rightarrow \quad E = \beta N k T$$

b) Determine an expression for the pressure of the gas and use this to obtain the pressure equation of state, i.e. $P = P(V, T)$.

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_{E,N} = \frac{Nk}{V} \quad \Rightarrow \quad PV = NkT$$

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Question 5

An ideal gas undergoes a isothermal expansion. Which of the following (choose one) is true regarding the change in entropy of the gas, ΔS, in this process?

i) $\Delta S = 0$ for both monoatomic and diatomic gases.
ii) $\Delta S > 0$ for both monoatomic and diatomic gases.
iii) $\Delta S < 0$ for both monoatomic and diatomic gases.
iv) Whether $\Delta S = 0$, or $\Delta S > 0$ or $\Delta S < 0$ depends on whether the gas is monoatomic or diatomic.

Briefly explain your answer

$dE = \delta Q + \delta W$ $\qquad$ $\delta Q = T dS \Rightarrow dS = \frac{1}{T}\delta Q$

For isothermal and ideal gas

$dE = 0$

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$\Rightarrow \delta Q = -\delta W$

For expansion $\delta W < 0 \Rightarrow \delta Q > 0$

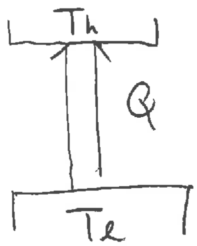
$\Rightarrow dS = \frac{1}{T}\delta Q > 0$

Question 6

Answer either part a) or part b) for full credit for this problem.

a) Use the second law of thermodynamics to show any process whose only result is the transfer of heat from a lower to a higher temperature reservoir is impossible.

suppose possible



$$\Delta S_{tot} = \Delta S_{high} + \Delta S_{low}$$

$$\left.\begin{aligned} \Delta S_{high} &= \frac{Q}{T_{high}} \\ \Delta S_{low} &= -\frac{Q}{T_{low}} \end{aligned}\right\} \Rightarrow \Delta S_{tot} = Q\left(\frac{1}{T_h} - \frac{1}{T_\ell}\right)$$

since $T_h > T_\ell$

$\Delta S_{tot} < 0$

impossible

b) The Helmholtz free energy is $F = E - TS$. Use this to show that

$$P = -\left(\frac{\partial F}{\partial V}\right)_S$$

and

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T.$$

$$dF = dE - TdS - SdT$$

$$= \cancel{TdS} - PdV - \cancel{TdS} - SdT$$

$$dF = -PdV - SdT$$

$$P = -\left(\frac{\partial F}{\partial V}\right)_T \qquad S = -\left(\frac{\partial F}{\partial T}\right)_V$$

second derivs equal $\Rightarrow \left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$

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