

## Statistical and Thermal Physics: Class Exam I

12 March 2019

Name: \_\_\_\_\_

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### Instructions

- There are 6 questions on 9 pages.
- Show your reasoning and calculations and always explain your answers.

### Physical constants and useful formulae

$$R = 8.31 \text{ J/mol K} \quad N_A = 6.02 \times 10^{23} \text{ mol}^{-1} \quad k = 1.38 \times 10^{-23} \text{ J/K} \quad 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$$

### Question 1

A quantity of air, a diatomic gas ( $\gamma = 7/5$ ), initially occupies a volume of  $2.0 \times 10^{-3} \text{ m}^3$  at atmospheric pressure and at temperature 300 K. This is allowed to expand adiabatically, doubling its volume. Determine the pressure and temperature at the end of this process.

## Question 2

Answer either part a) or part b) for full credit for this problem.

- a) Consider a system for which the energy is  $E = \alpha VT^4$  and  $P = \frac{1}{3} \alpha T^4$ . Determine expressions for

$$\left(\frac{\partial E}{\partial V}\right)_P \quad \text{and} \quad \left(\frac{\partial E}{\partial P}\right)_V.$$

Question 2 continued ...

b) The enthalpy of a gas is  $H = E + PV$ . Starting with the infinitesimal version of the first law  $dE = \delta Q + \delta W$  show that the heat capacity at constant pressure is

$$c_P = \left( \frac{\partial H}{\partial T} \right)_P.$$

### Question 3

A monoatomic ideal gas undergoes a cyclic process with the following stages, starting at initial pressure  $P_i$  and volume  $V_i$ . First it expands, during which  $P = V$  at all times, until the volume reaches three times the initial volume. Second, the pressure drops back to the original pressure while the volume remains constant. Finally it is compressed back to its initial volume while the pressure remains constant.

a) Sketch the process on a  $PV$  diagram.

b) Determine an expression for the work done in each stage.

Question 3 continued ...

c) Determine an expression for the heat added in each stage.



### Question 5

Answer either part a) or part b) for full credit for this problem.

a) The entropy of a monoatomic ideal gas is

$$S(T, V) = \frac{3}{2}Nk \ln(T) + Nk \ln(V) + g(N)$$

where  $g(N)$  is an unknown function of  $N$ . This gas undergoes a free expansion. Determine whether  $\Delta S$  is positive, negative or zero. Determine whether the change in Gibbs free energy is positive, negative or zero.

Question 5 continued ...

b) Use the Helmholtz free energy to show that, for a gas,

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T.$$

### Question 6

An ideal gas undergoes a compression at constant pressure.

- a) Which of the following is true regarding the change in entropy of the gas?
- i)  $\Delta S_{\text{gas}} = 0$ .
  - ii)  $\Delta S_{\text{gas}} < 0$ .
  - iii)  $\Delta S_{\text{gas}} > 0$ .
- b) Which of the following is true regarding the magnitude of the change in entropy of the environment?
- i)  $|\Delta S_{\text{env}}| = |\Delta S_{\text{gas}}|$ .
  - ii)  $|\Delta S_{\text{env}}| < |\Delta S_{\text{gas}}|$ .
  - iii)  $|\Delta S_{\text{env}}| > |\Delta S_{\text{gas}}|$ .

**Briefly explain your answers.**

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