Laboratory 13: Radioactive Decay (Simulation)

Some atoms have the remarkable property of being unstable, and decay into other elements by emitting different particles associated with different decay processes. Alpha, beta, and gamma decay lead to very different radioactive particles being emitted by the radioactive element, but in alpha and beta decay the decay process transforms the decaying element into another element. In addition, because the radioactivity is dependent on the number of radioactive atoms, and the probability of a radioactive event occurring, the mathematical description of the population of radioactive atoms is the same. The purpose of this lab is to model radioactive decay using ordinary six-sided dice or a suitably configured random number generator.

Within a population of radioactive particles N, the number of radioactive particles that actually decay ΔN in a time interval Δt is given by

$$-\frac{\Delta N}{\Delta t} = \lambda N \tag{1}$$

where λ is known as the decay constant, related to the probability of the particle to decay. Performing some calculus on Eq. (1) gives an exponential relationship between the number of particles remaining in a population of radioactive particles N and the original population N_0, given by

$$N = N_0\, e^{-\lambda t} \tag{2}$$

where t is the elapsed time from the original population.

1 Theory and Experimental Design

Usually in this experiment, the class will be rolling a large number of dice to simulate a large population of radioactive atoms. If the result of any die is a 6, it "decays" and is removed from the population. Instead of rolling dice we will generate equivalent events using a random number generator to generate integers between 1 and 6.

a) What is the decay constant of any individual die? Suppose that you throw the dice once every second. The decay constant can be determined from Eq. (2) by substituting in the fraction of dice that you expect to remain after a single throw of the dice, N/N_0. Use this to determine the decay constant.

b) Often when discussing radioactive elements, the term half-life appears. One half-life is defined as the time necessary for a population of radioactive elements to decay to one half of the original population. Determine the half-life (in units of number of throws of remaining dice) for the radioactive dice.

Here are some useful mathematical relationships involving the natural logarithm:

$$\ln e^a = a \tag{3}$$
$$\ln a^{-1} = -\ln a \tag{4}$$
$$\ln (ab) = \ln a + \ln b \tag{5}$$

c) You will be counting the number of remaining, undecayed, dice after every throw of the remaining dice and removing any "decayed" dice. Predict and sketch what you expect a graph of the remaining, undecayed, dice versus time (in units of throws of the dice) to look like. Is this graph linear?

d) Because a linear fit to data is desirable for determining physical quantities from measurements and comparing to theoretical predictions, linearize Eq. (2) such that you have a linear relationship in the form $y = mx+b$. You will likely need to use the aforementioned mathematical relationships involving the natural logarithm. Once you have the linearized equation, answer the following questions:

   i) What will you plot on the y-axis of your linearized graph?
   ii) What will you plot on the x-axis of your linearized graph?
   iii) What should the slope of your graph be?
   iv) What should the y-intercept of your graph be? **You won't be able to plug in an actual number until you begin the experiment with the entire class.**

2 Experiment – Radioactive Dice/Random Numbers

Because statistical experiments such as radioactivity rely upon large populations, we need to roll a large number of dice or generate a large number of random numbers using Excel.

a) Open `Excel` and enter the formula =RANDBETWEEN(1,6) into a cell. This will generate a random number integer in the range $1 \rightarrow 6$. Copy the formula into adjacent cells so that you produce 200 random numbers.

b) One can count the number of times that the number 6 occurs using =COUNTIF(RANGE,"'6"'). Use this to record the number of occurrences of 6 and then the number of dice, N that remain after the first "roll" (call this roll number 1).

c) Remove the numbers that returned 6, leaving their slots blank and repeat the procedure)call this roll number 2).

d) Repeat this for about 15 to 20 iterations.

e) Use `Excel` to create a graph N versus roll number. Does the graph agree with your prediction from part 1.c)? Print out the graph and attach it to your work.

f) By examining the graph of N versus roll number, estimate the half-life of the radioactive dice? Does this agree with your prediction from part 1.b)?

g) Use `Excel` to perform any necessary calculations so that you can create the linearized graph you identified in part 1.d). Is your graph actually linear? Have `Excel` fit a trendline to the data and record the slope and y-intercept of this. Print out the graph and attach it to your work.

h) Calculate percent differences for both your measured decay constant λ and y-intercept by comparing with the theoretical values using

$$\frac{|E-K|}{K} \times 100\% \tag{6}$$

where E is the experimental, measured value determined from the linear regression and K is the known, theoretical value.

3 Conclusions

a) Carbon-14 is produced when cosmic rays interact with the upper atmosphere and because of the carbon cycle, all living plants and animals have a relatively stable carbon-14 to carbon-12 ratio. Once an organism dies, the carbon-14 within begins to decay. The known half-life of carbon-14 is 5,730 years. If an unearthed fossil contains 15% of the expected carbon-14 in a living organism, how long ago did that creature die?