Laboratory 13: Buoyancy and Density

Archimedes reasoned that an immersed object experiences the same supporting buoyant force as the body of liquid it displaced. Thus the magnitude of the buoyant force equals the weight of the displaced fluid,

$$F_B = \overbrace{\rho_f V}^{\text{mass}} g$$

where ρ_f the density of the fluid in which the object is immersed, V the displaced volume and g the acceleration due to gravity. Measuring the buoyant force on an object can give information about the density of the fluid in which it is immersed, or the density of the object. The purpose of this laboratory will be to investigate the buoyant force and use it to determine densities.

1 Theory

a) Show that the weight of an object with density ρ and volume V is

$$W = \rho V g. \tag{1}$$

b) Use this result to determine an expression for the ratio of the weight to the buoyant force ,

$$\frac{W}{F_B}.$$

Use the resulting expression to determine an expression for the density of the object

$$\rho = ? \tag{2}$$

in terms of the density of the fluid ρ_f, the weight of the object W and the buoyant force F_B. Does the volume of the object matter here?

The object will be suspended at rest by a string and it will be possible to measure the tension in the string. The purpose of the following parts is to rewrite Eq. (2) in terms of measurable tensions.

c) Suppose that the object is suspended at rest in air. Determine an expression for the tension in the string,

$$T_{\text{in air}} = ? \tag{3}$$

in terms of the weight of the object. This gives a method for determining W.

d) Now suppose that the object is suspended from a string and immersed in a fluid. Determine an expression for the tension

$$T_{\text{in fluid}} = ?$$

in terms of the buoyant force, F_B and the weight of the object, W. Use this to show that the buoyant force is

$$F_B = T_{\text{in air}} - T_{\text{in fluid}}. \tag{4}$$

This gives a method for determining the buoyant force F_B.

2 Experiment: Density of an Immersed Object

a) Suspend a block of aluminum and measure the tension in the string, $T_{\text{in air}}$, when in air and, $T_{\text{in fluid}}$, when in water ($\rho_w = 1000\ \text{kg/m}^3$). Determine the weight and the buoyant force using Equations (3) and (4) and then use the equation you derived above (Equation (2)) to deduce the density of the aluminum block.

b) Determine the density of the aluminum directly by measuring its volume and mass and using $\rho = m/V$.

c) Determine the percentage difference between the two densities that you computed for aluminum.

d) Repeat the entire procedure for an object made of a different material that the instructor will provide.

3 Exercises

a) Consider a brass cylinder that is suspended in air. Air is also a fluid, with density $\rho_{\text{air}} = 1.29 \times 10^{-3}\ \text{g/cm}^3$ and yet you seldom consider the buoyant force exerted by air when you deal with an object like this. To see why determine the ratio, F_B/F_{grav}, of the buoyant force exerted by the air on the cylinder to the gravitational force exerted on the cylinder, whose density can be assumed to be $8.5 \times 10^3\ \text{kg/m}^3$. Is the buoyant force exerted by the air on the brass cylinder significant compared to the gravitational force exerted on the cylinder?

   Hint: The answer does not depend on the volume of the cylinder. Try to do some algebraic manipulations to eliminate the volume; if these fail, select a numerical value for the volume and do the relevant numerical calculations.

b) Consider two objects, each of the *same volume* and which are rigidly attached together. One has density exactly half that of water and the other exactly double that of water. The objects are initially held at rest beneath the surface of the water. Will they subsequently sink or rise and partially protrude above the surface? Explain your answer.

   Hint: Since the two objects are rigidly attached together, this can be assessed by considering all forces acting on each of the two objects. Try comparing the magnitudes of the total force pushing up to that of the total force pulling down.