Laboratory 9: Work and Kinetic Energy

Newton's Laws of motion relate forces and accelerations and, in principle, using these suffices for analyzing any classical mechanics situation. However, it is often more feasible and informative to use work and energy; in some cases this offers the only analytical method of describing the situation. These exercises introduce you to the concepts of work and energy.

1 Work and kinetic energy

Consider an object which moves under the influence of a constant force $\vec{\mathbf{F}}$ while moving in a straight line through displacement $\Delta\vec{\mathbf{r}}$, as illustrated in Fig. 1.

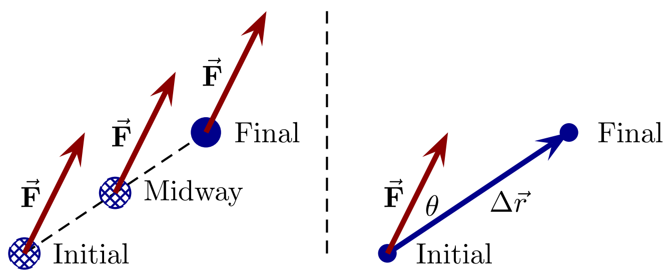

Figure 1: Constant force acting on an object moving in a straight line.

The work done by this force is

$$\boxed{W = F\Delta r\cos\theta} \tag{1}$$

where θ is as defined in Fig. 1, F is the magnitude of $\vec{\mathbf{F}}$ and Δr is the distance traveled by the object.

a) A block on a frictionless surface moves right while a hand pushes horizontally to the right on it. Using Newton's second law, explain whether the block's speed increases, decreases or is constant while the hand pushes on it.

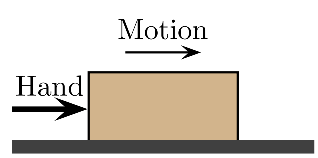

Figure 2: Block moving right.

b) Sketch vectors for the displacement and the force exerted by the hand on the block. Use the sketch to identify the angle between the vectors and then use the definition of work, Eq. (1), to explain whether the work done on the block by the hand is positive, negative or zero.

c) Use Eq. (1) to explain whether the work done on the block by Earth's gravitational force and also the work done by normal force are positive, negative or zero.

d) The net work, W_{net}, is the sum of the works done by all forces. Explain whether the net work done on the block is positive, negative or zero.

The net work done on any object must be related to changes in the state of motion of the object. The connection will be provided via the kinetic energy of the object. The kinetic energy of an object with mass m, which moves with speed v is

$$\boxed{K = \frac{1}{2}\, mv^2.} \tag{2}$$

The change in kinetic energy is $\Delta K = K_f - K_i$ where K_i is the kinetic energy at an earlier moment and K_f is the kinetic energy at a later moment.

e) Use your answer from section 1.a) to explain whether the change in kinetic energy, ΔK, of the block is positive, negative or zero.

The *work-kinetic energy theorem* states that

$$\boxed{\Delta K = W_{\text{net}}} \tag{3}$$

and this provides the key relationship between work and kinetic energy in any mechanical situation.

f) For the situation of Fig. 2, consider whether ΔK and W_{net} are positive, negative or zero. Do they agree as the work-kinetic energy theorem predicts?

g) A block on a frictionless surface moves left while a hand pushes horizontally to the right on it. Using Newton's second law explain whether the block's speed increases, decreases or is constant while the hand pushes on it.

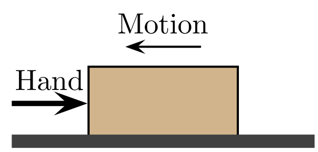

Figure 3: Block moving left.

Explain whether ΔK positive, negative or zero for this case.

Using Eq. (1), explain whether W_{net} is positive, negative or zero. Explain whether your answer agrees with what the work-kinetic energy theorem predicts.

h) In several cases illustrated in Fig. 4, two forces push horizontally on a block moving along a frictionless surface.

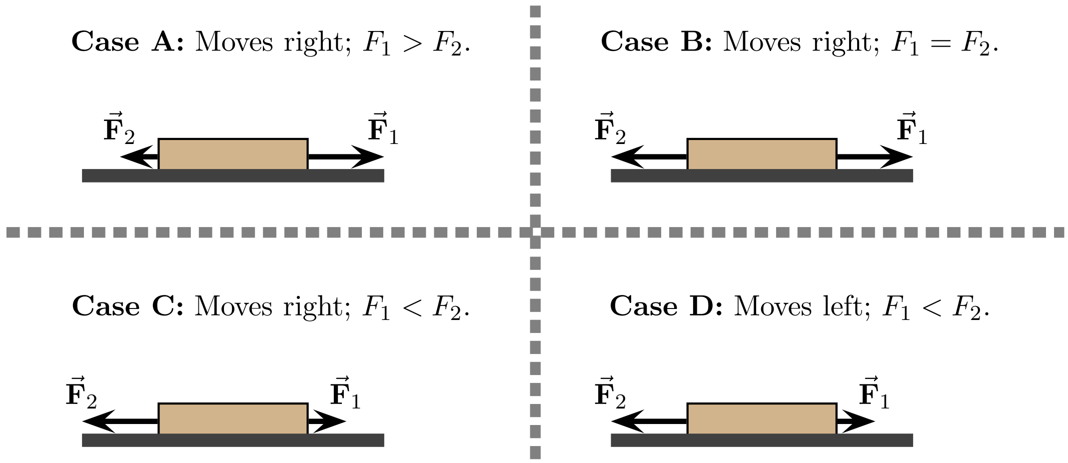

Figure 4: Blocks moving horizontally.

For each case use Newton's second law to describe whether the acceleration points left, right or is zero. Use this to describe whether the block's speed increases, decreases or is constant and use that to describe whether, ΔK, is positive, negative or zero. Enter these in the table below.

Case	Acceleration	Speed increases, decreases or constant?	ΔK
A			
B			
C			
D			

For each case, describe the signs of the work done by each force acting on the block, and use these to describe the sign of W_{net}. Enter these in the table below. Do your results agree with the work-kinetic energy theorem?

Case	W_1	W_2	W_{net}	Agree with work-KE theorem?
A				
B				
C				
D				

i) Consider two identical blocks that slide rightward along frictionless horizontal surfaces. A rope exerts a force on each block and the magnitudes of these are identical but their directions are different, as illustrated in Fig. 5.

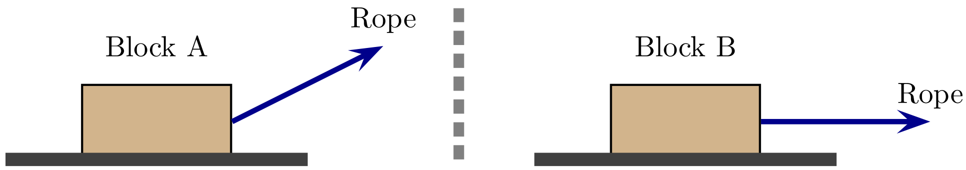

Figure 5: Forces at different angles on two blocks.

Using Newton's second law, explain whether the acceleration of the block A is equal to, larger than or smaller than that of block B.

Each block starts from rest and the two travel the same distance. Using the answer above and kinematics, explain whether ΔK for block A is equal to, larger than or smaller than that of block B.

Using Eq. (1) explain whether W_{net} for block A is equal to, larger than or smaller than that for block B. Then use the work-kinetic energy theorem to explain whether ΔK for block A is equal to, larger than or smaller than that of block B. Does this agree with your previous answer?

2 Applying the work-energy theorem: ascending and descending objects

A phone sits on the floor of an elevator as illustrated. The elevator moves vertically with rope always taut (i.e. not in free fall). Consider the following circumstances where the elevator moves through the same distance:

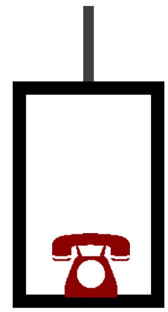

Figure 6: Phone in an elevator.

Case A: The elevator moves up at constant speed.

Case B: The elevator moves up with decreasing speed.

Case C: The elevator moves up with increasing speed.

Case D: The elevator moves down with decreasing speed.

Case E: The elevator moves down with increasing speed.

a) In each case, explain whether the work done by gravity, W_{grav}, is positive, negative or zero. Explain whether the work done by the normal force exerted by the floor on the phone, W_{n}, is positive, negative or zero. Use the information about the motion to describe whether ΔK is positive, negative or zero.

Enter the results into the table below.

Case	W_{grav}	W_{n}	ΔK
A			
B			
C			
D			
E			

b) In each case, use Newton's Second Law to describe whether the normal force is the same as, smaller than or larger than the weight. Use this to describe whether W_{net} is positive, negative or zero. In each case, are your results consistent with ΔK?

c) In each of the following cases the elevator moves up and the magnitudes of the forces are observed with the following results.

Case F: The magnitude of the normal force is larger than the magnitude of the gravitational force.

Case G: The magnitude of the normal force is smaller than the magnitude of the gravitational force.

Case H: The magnitude of the normal force is the same as the magnitude of the gravitational force.

For each case, describe whether each of $W_{\text{grav}}, W_{\text{n}}, W_{\text{net}}$ over the course of the motion is positive, negative or zero. Use your answer to explain whether the phone speeds up or slows down.

For each of the above cases explain what Newton's second law would predict for the speed of the phone as time passes. Does this agree with what you predicted using work?