Phys 311 2020

Lecture 36

Fri: Exam II

* Covers: Ch 2.5, 3.1, 3.4, 5.1 → 5.4

  Lectures 20 – 33

  HW 13 – 21

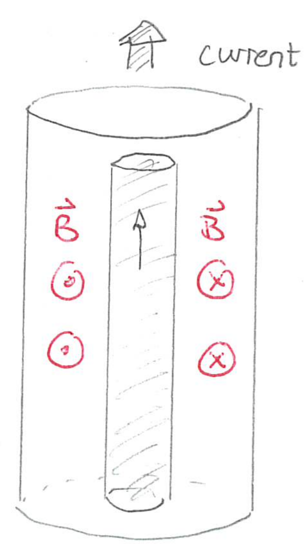
* Bring – Calculator
  – original eqn sheet
  – New ½ letter single side

* Study 2017 Class 2 (1.25 hr)
  2019 Class 2 (50 min)

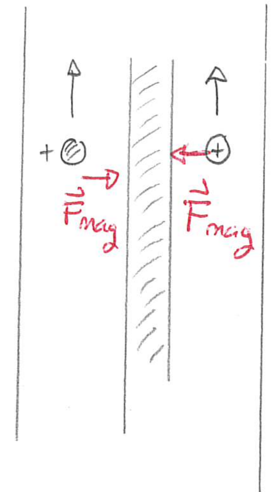
Currents in Ohmic Materials: Hall Effect

We now explore the details of a current that flows through a material and determine the extent to which a material can truly be Ohmic. Recall that, for an Ohmic material, the current density $\vec{J}$ and electric field $\vec{E}$ are related by $\vec{J} = \sigma \vec{E}$.

Consider a cylinder with a steady current along the length of the cylinder. Then a shaded column of current within the material will produce a magnetic field as illustrated.

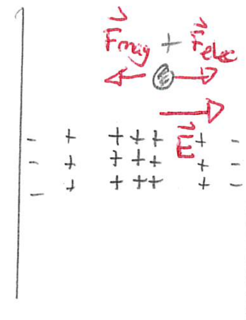


Now consider moving charges in the region beyond the shaded column. The magnetic field from the interior will exert a magnetic force on these charges. This force points radially inward for positive charges and radially outward for negative charges. This will produce a radial charge separation within the conductor. The results are



1) There will be a radial component to the electric field

2) In equilibrium there will be an electrostatic force that counterbalances the magnetic force.



3) If $\vec{J}$ is axial along the cylinder then

$$\vec{J} \neq \sigma \vec{E}$$

because $\vec{E}$ has a radial component

4) Inside the material we cannot have $\rho = 0$, which is a requirement for Ohmic materials in a steady state.

Thus perfect Ohmic materials do not exist. One can do an exact calculation and show that for typical currents ρ is very small and the material is nearly Ohmic

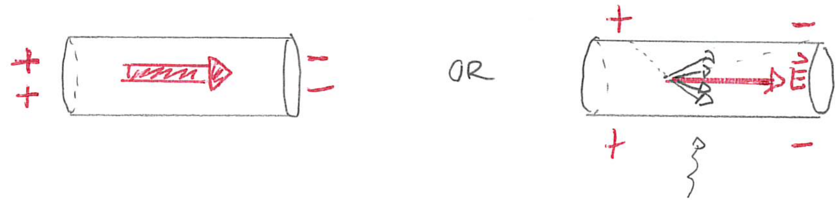
See: Zangwill Ch9 Prob 9.3

This is a variant of the Hall effect.

Electromotive force

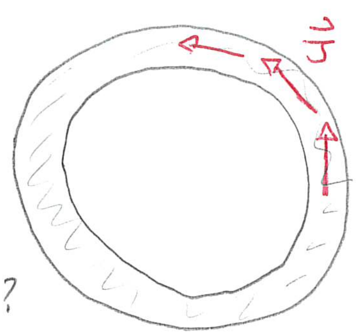
Ignoring the small Hall effect, we consider a perfect Ohmic material and consider the possibility that the current is sustained by an electrostatic field. First consider a cylindrical Ohmic object where $\vec{J}$ is along the cylinder axis. Inside the material $\rho = 0$

and therefore the only source of the necessary electric field could be only the surface. This could be produced by source charges at either end or on the curved surface



individual fields

Either of these works for a segment. Now consider a closed loop. Could either of these work. The first could not since it would require $\rho \neq 0$ inside. Perhaps something like the second could. To analyze this consider a circular loop with uniform thickness. Then assume



$$\vec{J} = J(s)\,\hat{\phi}$$

Can such a current density arise in a loop from an electrostatic field?

We know that an electrostatic field must satisfy. $\vec{\nabla} \times \vec{E} = 0$.

Then with

$$\vec{J} = \sigma \vec{E} \implies \vec{E} = \frac{1}{\sigma}\vec{J} = \frac{J(s)}{\sigma}\hat{\phi}$$

Now

$$\vec{\nabla}\times\vec{E} = \left[\frac{1}{s}\overset{0}{\cancel{\frac{\partial E_z}{\partial \phi}}} - \cancel{\frac{\partial E_\phi}{\partial z}}\right]\hat{s} + \left[\cancel{\frac{\partial E_s}{\partial z}} - \cancel{\frac{\partial E_z}{\partial s}}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sE_\phi) - \cancel{\frac{\partial E_s}{\partial \phi}}\right]\hat{z}$$

$$= \frac{1}{s}\frac{\partial}{\partial s}\left[sJ(s)/\sigma\right] = 0$$

→ boundary would create an issue??

Except in the special case where $J(s) = \frac{\text{const}}{s}$ this is not possible. We can eliminate this via a line integral around the loop. Then

$$\oint_{\text{loop}} \vec{E}\cdot d\vec{l} = \frac{1}{\sigma}\oint_{\text{loop}} \vec{J}\cdot d\vec{l} = \frac{J(s)}{\sigma}\underbrace{\oint_{\text{loop}} \hat{\phi}\cdot d\vec{l}}_{\neq 0}$$

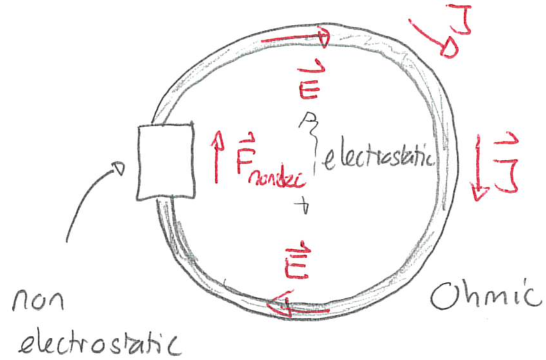
This would require $\oint \vec{E}\cdot d\vec{l} \neq 0$ and that is impossible for an electrostatic field. Thus

> In any current carrying closed loop there must be a segment where non-electrostatic forces act of the charge carriers in the current (i.e. forces not of the type $\vec{F} = q\vec{E}$ for some electrostatic field $\vec{E}$)

Thus there must be a non-electrostatic force present at some point in the loop. We will see that there are various possible sources of such forces:

1) forces provided by batteries
2) Forces provided by magnetic fields.

We consider a generic example in which a portion of the Ohmic loop is interrupted by a region in which a non-electrostatic force is present



We will show that:

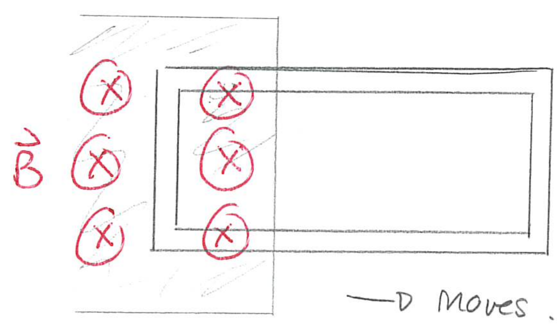
The non-electrostatic force satisfies

$$\vec{F}_{non-elec} = q \vec{f}_s$$

where q is the charge of any charge carrier in the current

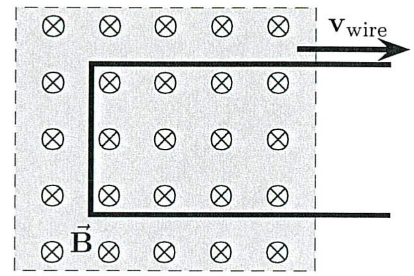
$\vec{f}_s$ is independent of q

We can illustrate this first via an example where a portion of a loop is dragged through a magnetic field



1 Current propulsion via a magnetic field

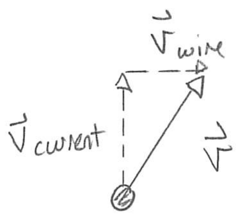
The illustrated wire partly resides in a region consisting of uniform magnetic field. The wire is dragged with velocity $\mathbf{v}_{\text{wire}}$ as illustrated.



a) Consider a particle with charge q in the left section of the wire. Suppose that, in a steady state it moves with velocity $\mathbf{v}_{\text{current}}$ along the wire. Determine an expression for the force acting on the particle. → relative to wire.

b) Determine an expression for the component of the force per unit charge $\mathbf{f}_s$ along the current direction.

Answers: a)



$$\vec{v} = \vec{v}_{wire} + \vec{v}_{current}$$

$$\vec{F}_{mag} = q\,\vec{v} \times \vec{B}$$

$$= q\left[V_{wire}\hat{x} + V_{current}\hat{y}\right] \times \left[-B\hat{z}\right]$$

$$= -q V_{wire} B \underbrace{\hat{x}\times\hat{z}}_{-\hat{y}} - q V_{current} B \underbrace{\hat{y}\times\hat{z}}_{\hat{x}}$$

$$= -q V_{current} B\,\hat{x} + q V_{wire} B\,\hat{y}$$

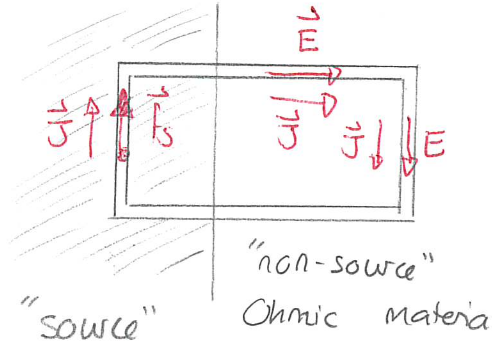
b) The component of the force along the current direction is

$$q V_{wire} B\,\hat{y}$$

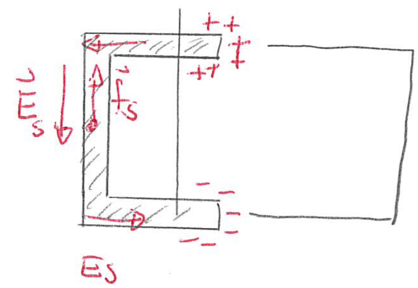
Denote this by $\vec{F}_{non\text{-}elec}$. Then

$$\vec{F}_{non\text{-}elec} = q\,\vec{f}_s \quad \text{where} \quad \vec{f}_s = V_{wire} B\,\hat{y}$$

We can regard the region of the magnetic field as a "source." Again we will consider the steady state. In this case the acceleration of the charge carriers is zero. Suppose that the source were separated from the rest of the material



The same effects would apply but there would now be an opposing electric field, denoted $\vec{E}_s$. In equilibrium the net force on any charge carrier is zero. Thus

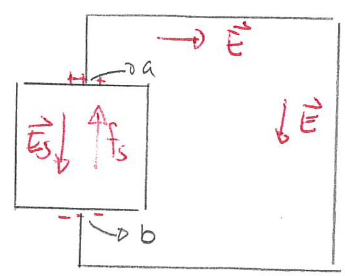


$$\vec{F}_{nonelec} + \vec{F}_{elec} = 0$$

$$\Rightarrow q\vec{f}_s + q\vec{E}_s = 0 \quad \Rightarrow \vec{E}_s = -\vec{f}_s$$

With an electric field of this type, around the loop we obtain

$$\oint \vec{E} \cdot d\vec{l} = 0$$



$$\int_{\text{source}} \vec{E}_s \cdot d\vec{l} + \int_{\text{wire } a \to b} \vec{E} \cdot d\vec{l} = 0$$

$$\Rightarrow -\int_{\text{source}} \vec{f}_s \cdot d\vec{l} = -\int_{\substack{\text{wire}\\ a \to b}} \vec{E} \cdot d\vec{l}$$

$$\Rightarrow \int_{\text{source}} \vec{f}_s \cdot d\vec{l} = \int_{\substack{\text{wire}\\ a \to b}} \vec{E} \cdot d\vec{l} = -\Delta V_{a \to b} = -\left[V(b) - V(a)\right]$$

Thus

$$V(a) - V(b) = \int_{\substack{\text{source} \\ a \to b}} \vec{f}_s \cdot \vec{dl}$$

So the force per unit charge acts as a "field" that contributes to the electrostatic potential around the circuit. Thus we define

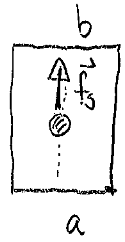
$$\boxed{\begin{array}{l}\text{The electromotive force produced by the source is} \\ \mathcal{E} = \int \vec{f}_s \cdot \vec{dl}\end{array}}$$

and

$$\boxed{\Delta V = V(a) - V(b) = \mathcal{E}}$$

Thus

Source produces force per unit charge, $\vec{f}_s$



EMF

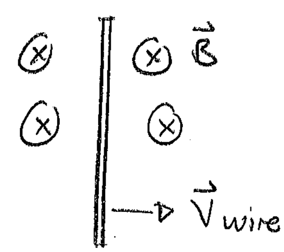
$$\mathcal{E} = \int_a^b \vec{f}_s \cdot \vec{dl} = V(b) - V(a)$$

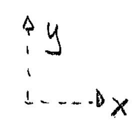
acts as a non-electrostatic version of the electrostatic potential

<u>Example:</u> A wire with length L is dragged perpendicular to a uniform magnetic field. Determine the EMF across the ends

<u>Answer:</u> Using the co-ordinates

$$\vec{f}_s = B v_{wire} \hat{y}$$





Using a path along the wire from $a \to b$

$$0 \leq y \leq L \qquad \vec{dl} = dy\, \hat{y}$$

$$\Rightarrow \vec{f}_s \cdot \vec{dl} = B v_{wire}\, dy \quad \Rightarrow \quad \int \vec{f}_s \cdot \vec{dl} = B v_{wire} L$$