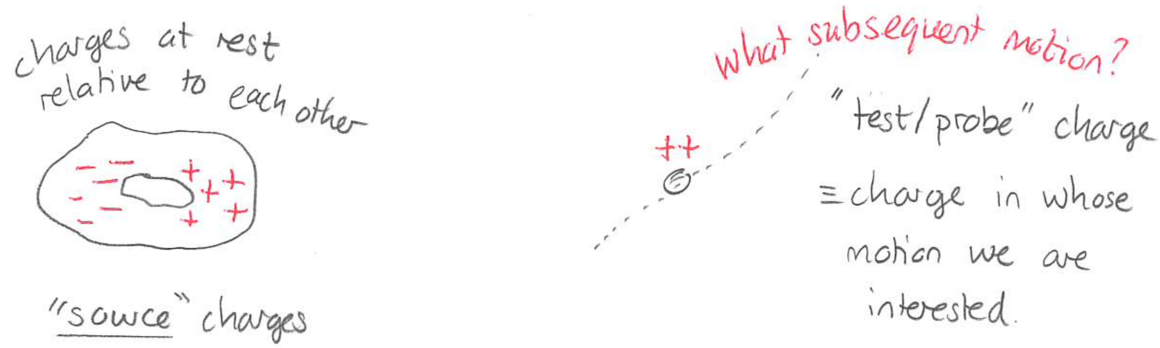
Fri: HW by 5pm.

Fri: Read 2.1.4, 2.2.1

Electrostatics

Electrostatics considers situations where a collection of stationary charges interacts with one other charged object



In classical physics we can address questions of motion in terms of velocity, acceleration and force. The crucial question is:

"What force does the collection of source charges exert on the test charge?"

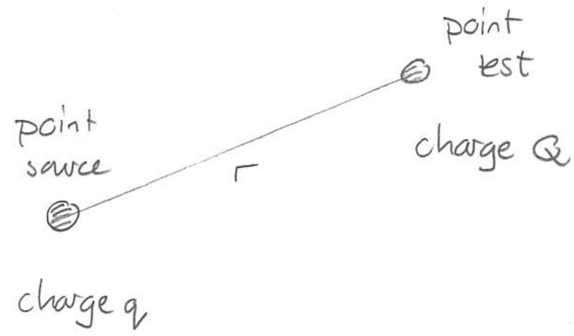
Provided that the source charges are stationary, electrostatics offers a framework for addressing this question. It does so in terms of an intermediary called an electric field. Much of electrostatics is the machinery needed to determine the electric field.

Demo: * PhET Electric Field Hockey
* Level 3

Coulomb's Law

The simplest situation in electrostatics is that where there is a single point source charge that interacts with a single point test charge. The fundamental law that describes this situation, Coulomb's Law, provides the basis for all of electrostatics.

The elementary version of this has two parts:



1) the magnitude of the force is

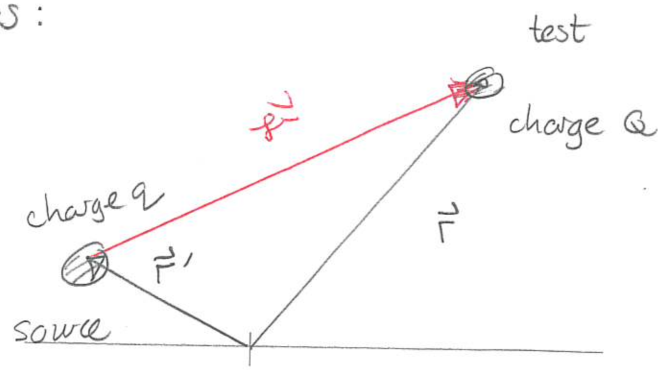
$$F = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2}$$

where r is the distance between the two charges

2) the direction is attractive or repulsive depending on whether the charges are like or unlike.

We need to reformulate this so that it captures both parts in a single mathematical statement that is amenable to algebra and calculus.

First we describe the situation as:



1) a source particle with charge q is located at position $\vec{r}\,'$

2) a test particle with charge Q is located at position $\vec{r}$.

The separation vector from the source to the test is

$$\boxed{\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'}$$

Then Coulomb's Law states.

> The force exerted by the source charge on the test charge is:
>
> $$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{\mathscr{r}^2} \hat{\mathscr{r}}$$
>
> where
>
> $$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$$
>
> is the permittivity of free space

Note that

$\mathscr{r} \equiv$ magnitude of $\vec{\mathscr{r}}$ separation vector

$\hat{\mathscr{r}} \equiv$ unit vector in direction of $\vec{\mathscr{r}}$

Then standard vector algebra gives

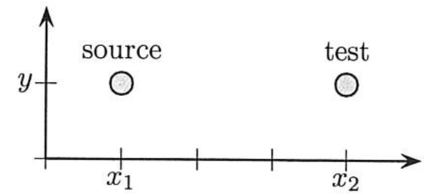
$$\hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{\mathscr{r}}$$

and thus

$$\frac{1}{\mathscr{r}^2}\hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{\mathscr{r}^3}$$

1 Coulomb's law

A source partiocle with charge q and a test particle with charge Q are located as illustrated. Using the framework of position and separation vectors, determine an expression for the force that the source exerts on the test in the illustrated situation. The result must depend on x_1, x_2 and perhaps y. You must explicitly write the two position vectors in terms of components and use them to carry out the algebra. Verify that this is consistent with facts that you have learned about Coulomb's law previously.



Answer:

$$\vec{r} = x_2\hat{x} + y\hat{y}$$

$$\vec{r}\,' = x_1\hat{x} + y\hat{y}$$

describes situation

+

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' = x_2\hat{x} + y\hat{y} - (x_1\hat{x} + y\hat{y})$$

$$= (x_2 - x_1)\,\hat{x}$$

$$\Rightarrow \mathscr{r} = (x_2 - x_1)$$

So
$$\hat{\mathscr{r}} = \frac{(x_2 - x_1)\,\hat{x}}{(x_2 - x_1)} = \hat{x}$$

entirely algebraic operations

Thus

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\,\frac{qQ}{(x_2 - x_1)^2}\,\hat{x}$$

ANSWER

Then $x_2 - x_1 \equiv$ distance between charges. So

elementary version

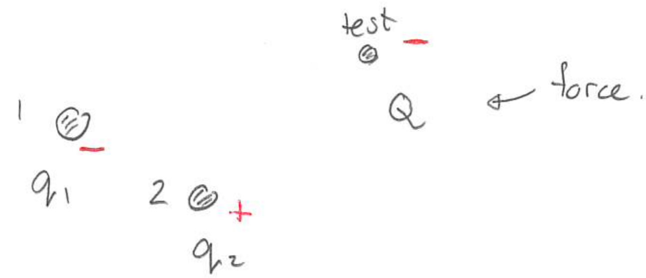
if charges are like $qQ > 0 \Rightarrow \vec{F}$ is $+\hat{x} \Rightarrow$ repulsive

if " " unlike $qQ < 0 \Rightarrow \vec{F}$ is $-\hat{x} \Rightarrow$ attractive.

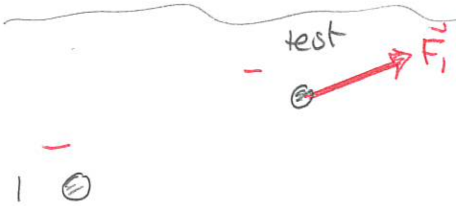
Magnitude is $\dfrac{1}{4\pi\epsilon_0}\,\dfrac{qQ}{(\text{distance})^2}$

Force exerted by multiple stationary sources: superposition

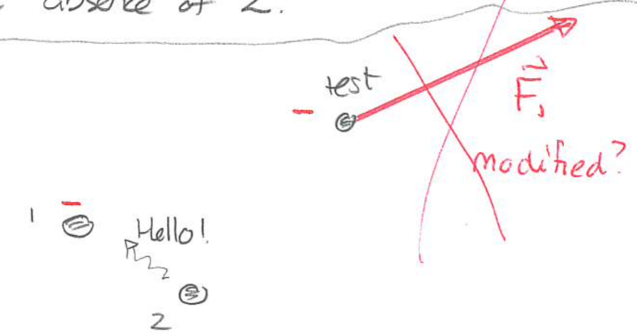
In general a test charge could be in the presence of multiple stationary source point charges. What force will the sources collectively exert on the test charge? Consider a situation where there are two source charges, labeled 1 and 2.



We can ask whether the force exerted by 1 is any different that it would have been in the absence of 2?



vs



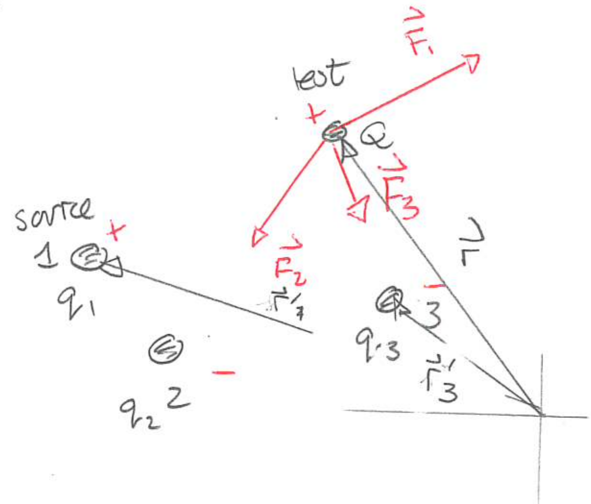
Source 2 absent
↳ Coulomb's law determines $\vec{F}_1$

Source 2 present - somehow modifies the force exerted by source 1 ??

Observational and experimental evidence indicates that this modification does not occur. So source 1 exerts the same force, determined by Coulomb's Law, whether or not any other sources are present. The same applies to other source charges.

Suppose that the sources are labeled

$i = 1, 2, 3, \ldots$



Denote the charge of source i by

q_i

Then let

$\vec{F}_i$ = force exerted by source i if none of the other sources were present.

We can calculate this using Coulomb's Law.

Now the previous observations are codified via the <u>superposition principle</u>, which states that the force exerted by any source is not altered by the presence of other sources. Specifically:

> The force exerted by a collection of stationary point sources is:
>
> $$\vec{F} = \sum_i \vec{F}_i$$
>
> where
>
> $$\vec{F}_i = \frac{1}{4\pi\epsilon_0} \frac{q_i Q}{\mathscr{r}_i^2} \hat{\mathscr{r}}_i \qquad \vec{\mathscr{r}}_i = \vec{r} - \vec{r}\,'_i$$
>
> is the force exerted by source i in the absence of other sources

Superposition principle

The separation vector arrangement is illustrated.