Phys 311 2020

Lecture 1

Intro *Syllabus

*Course website - main page

- HW list
- lecture materials page

Course structure

* meets in-person every day
* COVID rules - masks
  - seating
* HW Tues, Fri
* Exams

This week: Weds Read

Fri: HW 1 by 5pm

Electromagnetism Overview

Electromagnetism is the theory that describes how charged particles interact. It does this via intermediaries, called electric + magnetic fields and much of electromagnetic theory deals with the process of calculating these fields.

Demo: PhET Radio Waves + EM fields

1) hide fields + oscillate observe receiving charges
2) observe fields

Electromagnetic theory was originally developed within the framework of classical physics and refers to forces, accelerations and so on. The theory reached a complete mature form with the arrival of Maxwell's four equations in the late 19^{th} century. After quantum theory appeared it was adapted to describe situations involving charged particles that obey the laws of quantum theory.

The Phys 311 course will

1) provide a complete description of electric and magnetic fields and the role that they play

2) provide various techniques for computing electric and magnetic fields

3) arrive at Maxwell's equations which are the main rules used to determine electric and magnetic fields in all situations

You will need to develop or use

1) basic classical mechanics including Newton's Laws and conservation of energy

2) vector algebra, vector calculus

3) calculus in three dimensions

Vector algebra 1.1.1

The fundamental mathematical entities that are used in electromagnetic theory are vectors associated with:

- location of charges
- direction of currents
- electric and magnetic fields

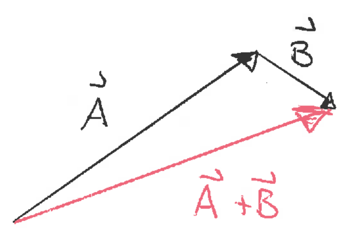
To manage these we need:

1) vector algebra ~ algebraic operations (addition + multiplication) between vectors

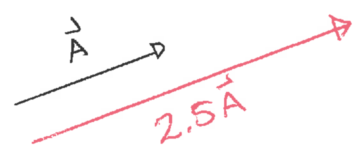
2) vector calculus ~ differentiation and integration adapted to three dimensional vector functions

We first consider basic vector algebra operations by illustrating these with displacement vectors in three dimensions. There are two basic operations associated with any set of vectors:

1) vector addition ~ combine successive displacements.



2) scalar multiplication ~ rescaled displacement.



The rigorous mathematical strategy for describing vectors involves abstracting properties from the intuitive displacement vectors

The basic ideas is that vectors form a set (called a vector space) with two operations that satisfy a specific set of properties.

Halmos "Finite dimensional vector spaces"

For any set of vectors there are two operations:

i) addition $\Rightarrow$ for $\vec{A}, \vec{B}$ the sum, $\vec{A} + \vec{B}$ is also a vector

ii) scalar multiplication $\Rightarrow$ for $\vec{A}$ and a scalar λ, $\lambda \vec{A}$ is a vector

These satisfy

1) addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

2) addition is associative: $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

3) there is a zero vector, $\vec{0}$, $\vec{A} + \vec{0} = \vec{A}$

4) for any vector, $\vec{A}$, there is an additive inverse, $-\vec{A}$ s.t. $\vec{A} + (-\vec{A}) = \vec{0}$

5) for any vector $\vec{A}$ and scalars, α, β $\alpha(\beta \vec{A}) = (\alpha\beta)\vec{A}$

6) for the scalar 1 and any vector $\vec{A}$ $1\vec{A} = \vec{A}$

7) for any vectors $\vec{A}, \vec{B}$ and scalar α: $\alpha(\vec{A} + \vec{B}) = \alpha\vec{A} + \alpha\vec{B}$

8) for any vector $\vec{A}$ and scalars α, β $(\alpha + \beta)\vec{A} = \alpha\vec{A} + \beta\vec{A}$

There are many examples of vector spaces:

1) column vectors $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

2) matrices $M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$

3) continuous functions

The theory of linear algebra then provides a framework that applies to all types of vectors.

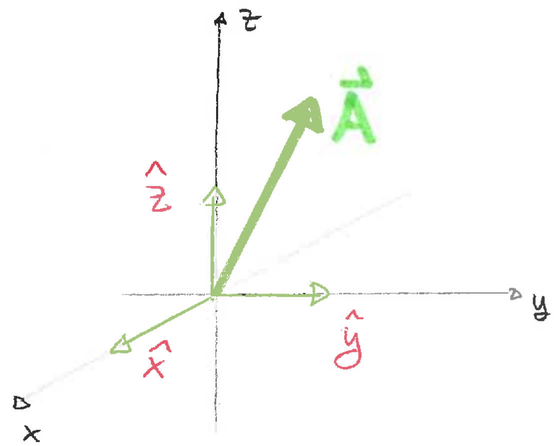
Vector bases and vector components 1.1.2

Consider a displacement vector in three dimensions, $\vec{A}$. It is intuitive that it can be expressed in terms of three special unit vectors along the Cartesian x, y, z axes. Specifically

vector

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

scalar



where A_x, A_y, A_z are scalars.

This possibility is assured via various theorems of linear algebra

> For any three dimensional vector, $\vec{A}$, there exist
>
> 1) three basis vectors $\hat{x}, \hat{y}, \hat{z}$
> 2) three <u>unique</u> scalars A_x, A_y, A_z
>
> such that
>
> $$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

The three scalars are called the components of $\vec{A}$ (in the basis $\{\hat{x}, \hat{y}, \hat{z}\}$).

If we use one particular fixed basis then the vectors can be expressed uniquely (in a 1-1 correspondence) with column vectors

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} \quad \Rightarrow \quad \vec{A} \leftrightarrow \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$\vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z} \quad \Rightarrow \quad \vec{B} \leftrightarrow \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$$

and

$$\vec{A} + \vec{B} \leftrightarrow \begin{pmatrix} A_x + B_x \\ A_y + B_y \\ A_z + B_z \end{pmatrix} \quad \text{and} \quad \alpha \vec{A} \leftrightarrow \alpha \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \alpha A_x \\ \alpha A_y \\ \alpha A_z \end{pmatrix}$$

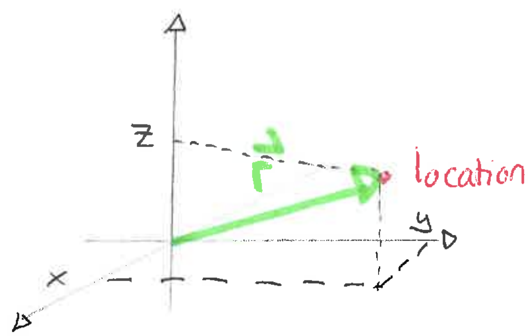
Position vectors 1.1.4

In electromagnetic theory we will use position vectors to describe

- locations of charged particles
- locations at which fields are to be calculated

The basic conceptual idea is that a position vector is a displacement from the origin to the location of interest

We denote such a position vector as follows:



vector   components/basis   column

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \longleftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

components

We can easily show that if we define addition and multiplication in the usual way via columns

$$\underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\vec{r}} + \underbrace{\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}}_{\vec{r}\,'} = \underbrace{\begin{pmatrix} x+x' \\ y+y' \\ z+z' \end{pmatrix}}_{\vec{r}+\vec{r}\,'}$$

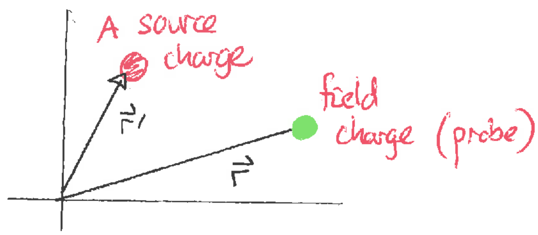
and

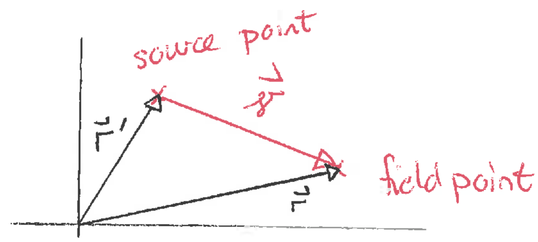
$$\alpha \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\vec{r}} = \underbrace{\begin{pmatrix} \alpha x \\ \alpha y \\ \alpha z \end{pmatrix}}_{\alpha\vec{r}}$$

We can show that this process of addition and multiplication produces vectors with an identity independent of the co-ordinate system

Separation vectors

We often need to consider the effects of one charge (source charge) on another (field charge). The location of the source charge is called the source point and the location of the field charge is called the field point





Let

$\vec{r} \equiv$ position vector to field point

$\vec{r}' \equiv$ " " " source point

Then, the separation vector between these is

$$\boxed{\vec{\mathscr{r}} = \vec{r} - \vec{r}'}$$

field pt (probe)

source pt (source charge)

Thus if

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$
$$\vec{r}' = x'\hat{x} + y'\hat{y} + z'\hat{z}$$

then

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}' = (x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}$$

1 Pyramid

A pyramid has a square base with sides of length L. The apex of the pyramid is a height h above the center of the base. The base lies in the first quadrant of the xy plane. Let A be the corner at the origin, B that along the x axis, C that away from either axis and D that along the y axis. Let E be the apex.

a) Determine expressions in terms of the standard basis vectors for the position vector of each corner.

b) Determine an expressions in terms of the standard basis vectors for the separation vector from B to C.

c) Determine an expressions in terms of the standard basis vectors for the separation vector from B to E.

d) Determine an expressions in terms of the standard basis vectors for the separation vector from C to E.

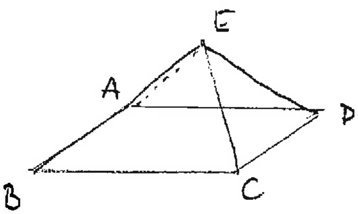
<u>Answer:</u> a) $\vec{r}_A = 0\hat{x} + 0\hat{y} + 0\hat{z}$

$\vec{r}_B = L\hat{x} + 0\hat{y} + 0\hat{z} = L\hat{x}$

$\vec{r}_C = L\hat{x} + L\hat{y}$

$\vec{r}_D = L\hat{y}$

$\vec{r}_E = \frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + h\hat{z}$



b) $\vec{\mathscr{r}}_{B\text{ to }C} = \vec{r}_C - \vec{r}_B$

$= L\hat{x} + L\hat{y} - L\hat{x} \Rightarrow \vec{\mathscr{r}}_{B\text{ to }C} = L\hat{y}$

c) $\vec{\mathscr{r}}_{B\text{ to }E} = \vec{r}_E - \vec{r}_B$

$= \frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + h\hat{z} - L\hat{x}$

$\vec{\mathscr{r}}_{B\text{ to }E} = -\frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + h\hat{z}$

d) $\vec{\mathscr{r}}_{C\text{ to }E} = \vec{r}_E - \vec{r}_C = \frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + h\hat{z} - (L\hat{x} + L\hat{y})$

$\Rightarrow \vec{\mathscr{r}}_{C\text{ to }E} = -\frac{L}{2}\hat{x} - \frac{L}{2}\hat{y} + h\hat{z}$