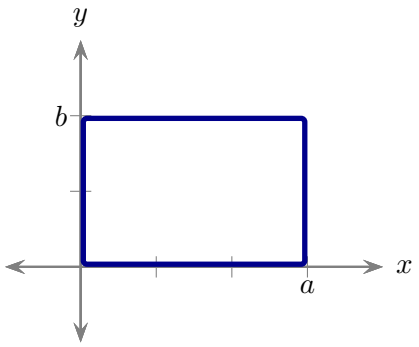
Electromagnetic Theory: Homework 17

Due: 23 October 2020

1 Force on a loop of wire

A rectangular loop lies in the xy plane and carries constant current, I, which flows counterclockwise.



a) An external source produces a magnetic field $\mathbf{B} = \alpha x \hat{\mathbf{z}}$ where $\alpha > 0$ is a constant with units of T/m. Determine the net force on the loop.

b) Suppose that the loop is placed in a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$. Without calculating exactly describe whether the net force on the loop is zero or not.

c) Suppose that the loop is placed in a uniform magnetic field $\mathbf{B}$ oriented in any direction. Is the net force on the loop zero or not?

2 Force on an arbitrary loop of wire in a uniform magnetic field

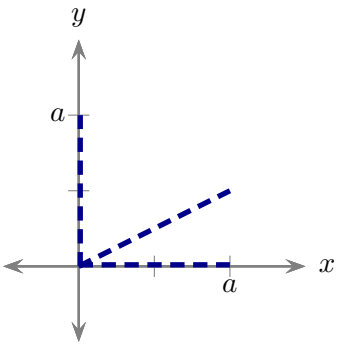
Consider a loop of wire with an arbitrary shape that carries a constant uniform current and is placed in a uniform magnetic field. Is it possible to arrange the shape of the wire relative to the field so that the net force on the loop is non-zero? Explain your answer. *Hint: A few lines of mathematics starting from the general formula for a force that a magnetic field exerts on a one-dimensional current can prove the result.*

3 Sheet of current on a flat surface

A surface on the xy plane carries a current with surface current density

$$\mathbf{K} = \beta\,(x\hat{\mathbf{y}} - y\hat{\mathbf{x}})$$

where β has units of A/m^2. Determine the current that flows across each of the dashed line segments.



4 Force on a rotating disk

A disk of radius R in the xy plane carries a uniform charge density σ and rotates about the z axis with angular velocity ω.

a) Determine the surface current density.

b) Suppose that the disk is in the presence of a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ where B is constant. Determine the net force that the field exerts on the disk.

c) Suppose that the disk is in the presence of a magnetic field $\mathbf{B} = B\sin(\phi/2)\hat{\mathbf{z}}$ where B is constant. Determine the net force that the field exerts on the disk.

5 Rotating sphere

A uniformly charged sphere with radius R and charge density ρ rotates about its z axis with angular velocity ω.

a) Determine an expression for the volume current density as a function of r, θ, ϕ.

b) Determine the current that flows through the yz plane where $z > 0$ and $y > 0$.

c) Determine the current that flows through the xz plane where $x > 0$.

6 Cylinder in a magnetic field

A cylinder has radius R and length L. Suppose that current flows from the axis of the cylinder radially outward toward the surface of the cylinder and assume that this is uniform along the length of the cylinder.

a) Let I be the total charge that flows through the outer curved surface of the cylinder. Determine the volume current density at the curved surface of the cylinder.

b) Assuming that no charge accumulates anywhere in the cylinder determine the current density at any any point inside the cylinder.

c) Suppose that the cylinder is placed in a uniform external magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ and is held at rest. The cylinder is released. Describe qualitatively the subsequent motion of the cylinder.