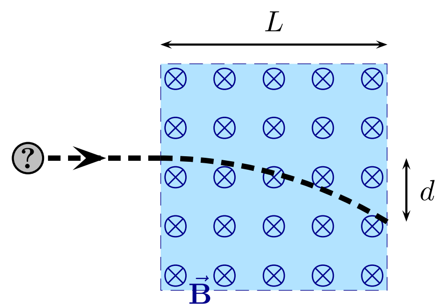
Electromagnetic Theory: Homework 16

Due: 20 October 2020

1 Circular motion of a particle in a constant field

A particle is fired into a region with constant magnetic field pointing into the page. The trajectory of the particle is as illustrated. The width of the field region is L and the particle is deflected by amount d at the point that it leaves the field.



a) Is the particle positive or negative? Explain your answer.
b) Determine an expression for the radius of orbit of the particle in terms of L and d.
c) Determine an expression for the momentum of the particle in terms of L, d, B and the charge q.

2 Particle in uniform electric and magnetic fields

A particle of charge Q and mass m is in uniform magnetic, $\mathbf{B} = B\hat{\mathbf{z}}$ and electric $\mathbf{E} = -E\hat{\mathbf{x}}$ fields. At $t = 0$, the particle's velocity lies in the xy plane.

a) Show that the particle's subsequent motion is restricted to the xy plane.
b) Use the Lorentz force law to obtain differential equations for the components of velocity. Solve these equations for all components of velocity.
c) Solve the resulting equations to give $x(t), y(t)$ and $z(t)$.
d) Suppose that the particle is originally at the origin with velocity $\mathbf{v}(0) = (E/B)\hat{\mathbf{y}}$. Determine expressions for and sketch the resulting trajectory.
e) Suppose that the particle is originally at the origin with velocity $\mathbf{v}(0) = (E/2B)\hat{\mathbf{y}}$. Determine expressions for and sketch the resulting trajectory.

3 Cycloid motion in crossed electric and magnetic fields

A particle with mass m and charge Q is in the uniform fields $\mathbf{E} = E\hat{\mathbf{z}}$ and $\mathbf{B} = B\hat{\mathbf{x}}$. If the particle is released from rest at the origin at $t = 0$ then the trajectory is given by (see Griffiths, pg 214-215)

$$y(t) = \frac{E}{\omega B}\left[\omega t - \sin \omega t\right]$$
$$z(t) = \frac{E}{\omega B}\left[1 - \cos \omega t\right]$$

where $\omega = QB/m$. We would like to consider the motion for limiting cases of electric and magnetic fields.

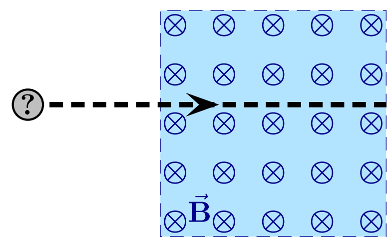
a) *Using physical reasoning* explain what the motion of the particle should be like if $E = 0$ but $B \neq 0$.

b) Check that the equations predict the motion correctly when $E = 0$ but $B \neq 0$.

c) *Using physical reasoning* explain what the motion of the particle should be like if $E \neq 0$ but $B = 0$.

d) Do the equations predict the motion correctly when $E \neq 0$ but $B = 0$? You should observe an issue with this. To resolve this expand $\sin \omega t$ and $\cos \omega t$ as Taylor series in ωt (you will only need to write down the first three terms to see the pattern), substitute the cyclotron frequency into the resulting expressions and then set $B = 0$. Do these predict the motion correctly?

4 Thomson's electron discovery experiment



The electron was first identified in an experiment in which the then unknown particles were fired into a region of constant magnetic field as illustrated. At the same time a uniform electric field was applied so that the particles moved through the field region in a straight line.

a) Assuming that the particles are negatively charged, in which direction must the uniform electric field point so that the net force on the particles is zero?

b) Suppose that the electric and magnetic fields can be measured accurately. If these are adjusted so that the net force is zero then describe how one could determine the speed of the particles by only using the electric and magnetic fields.

c) If electric field is turned off and the particle is fired into the magnetic field as illustrated it will move in a circle whose radius can be measured. Describe how one can use this to determine the ratio of the particle's charge to its mass. How would the result from the previous part enter?

In the experiment the charged particles were ejected from various metals and then accelerated into the fields. The result was that the charge to mass ratio was independent of the metal used, indicating the same type of particle in various different metals.