Electromagnetic Theory: Homework 15

Due: 16 October 2020

1 Multipole expansion for three dimensional distributions

a) Inside a sphere of radius R the charge density is

$$\rho(\mathbf{r}') = \frac{3q}{4\pi R^3}.$$

Sketch this qualitatively and determine the dipole and monopole moments of this. Use these to obtain an approximate expression for the potential produced by this distribution.

b) Inside a sphere of radius R the charge density is

$$\rho(\mathbf{r}') = \frac{3\alpha}{4\pi R^3}\ \sin\phi'.$$

Sketch this qualitatively and determine the dipole and monopole moments of this. Use these to obtain an approximate expression for the potential produced by this distribution.

2 Multipole expansion for one dimensional distributions

A one-dimensional circular loop of wire has radius R and lies in the xy plane. Charge is distributed along the loop (in spherical coordinates) according to

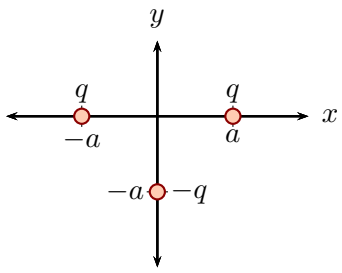
$$\lambda(\mathbf{r}') = \lambda \sin\left(\frac{\phi'}{2}\right)$$

a) Determine the monopole and dipole terms in the potential that this produces.

b) *This part is a **bonus** problem, worth a total of 10 bonus points.* Determine the quadrupole term in the expansion for the potential.

3 Multipole expansion for point charges

Several point charges are situated as illustrated. Each is distance a from the origin.



a) Determine an expression for the monopole and dipole moments of this distribution.

b) Use these moments to determine an approximate expression for the electrostatic potential produced by the distribution.

4 Electric dipole potential and field

A spherical shell contains charge which is distributed according to (in spherical coordinates)

$$\rho(\mathbf{r}') = \alpha \cos(\theta')$$

where $\alpha > 0$ is a constant.

a) Which of the following best represents the dipole moment? In the following $p > 0$.

i) $\mathbf{p} = p\hat{\mathbf{z}}$
ii) $\mathbf{p} = -p\hat{\mathbf{z}}$
iii) $\mathbf{p} = p\hat{\mathbf{x}}$
iv) $\mathbf{p} = -p\hat{\mathbf{x}}$
v) $\mathbf{p} = p\hat{\mathbf{y}}$
vi) $\mathbf{p} = -p\hat{\mathbf{y}}$

b) Are there any locations where the dipole potential satisfies $V_{\text{dipole}} = 0$? If so, describe them.

c) Are there any locations where the dipole electric field satisfies $\mathbf{E}_{\text{dipole}} = 0$? If so, describe them.

5 Force exerted by a dipole

A dipole is located at the origin and has dipole moment $\mathbf{p} = p\,\hat{\mathbf{x}}$.

a) Determine an expression, in spherical coordinates, for the electrostatic potential produced by the dipole.

b) Determine an expression, in spherical coordinates, for the electric field produced by the dipole.

c) A point particle with charge q is located at $(a, 0, 0)$ (in Cartesian coordinates). Determine the force exerted by the dipole on the charge.

d) A point particle with charge q and mass m is held at rest at $(a, 0, 0)$ (in Cartesian coordinates). The particle is released. Determine an expression for its speed when it is infinitely far from the origin.