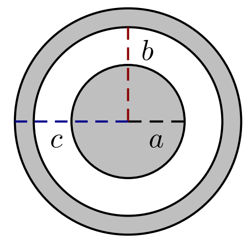
Phys 311
Fall 2020

Electromagnetic Theory: Homework 13

Due: 8 October 2020

1 Capacitance of a coaxial cable

An infinitely long coaxial cable consists of two concentric conducting cylinders, whose cross-section is as illustrated. The radii from inside to outside are $a < b < c$.



a) Determine the capacitance for a length L of the coaxial cable.

b) Does the capacitance depend on the outer radius of the outer cylinder?

c) Suppose that the inner cylinder were replaced by a hollow cylinder with the same radius. Would this change the capacitance of the coaxial cable? Explain your answer.

2 Capacitance of a single spherical conducting shell

The capacitance of a single spherical shell can be determined by first considering two concentric conducting shells, with the inner shell having radius a and the outer shell radius b. The capacitance of this can be determined as a function of a and b. The capacitance of a single shell is determined by taking the limit $b \to \infty$.

a) Determine the capacitance of a single spherical shell of radius a.

b) Determine the energy stored, using the capacitance, in a single charged shell of radius a if the shell carries a total charge q.

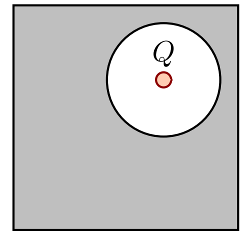
3 Energy in concentric spherical capacitors

Consider a capacitor that consists of a solid conducting sphere with radius a and a concentric conducting spherical shell with radius $b > a$.

a) Suppose that the charge on each sphere is multiplied by a factor of α. By what factor does the energy stored in the capacitor change?

b) Suppose that the charge on each sphere is held constant but that the radius of the outer sphere is increased. Does the energy stored increase, decrease or stay the same? Explain your answer.

4 Conductor with a spherical cavity



A conductor contains a spherical cavity with radius R. The conductor is initially neutral. Subsequently a point particle with charge Q is placed at the center of the cavity.

a) Gauss' law can be used to show that for any surface, the charge density on the surface, σ, is given by

$$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}$$

where the electric fields are those immediately above and below the surface and $\hat{\mathbf{n}}$ is the unit vector perpendicular to the surface pointing in the direction from below to above. Use this to determine the charge density on the inner surface of the cavity.

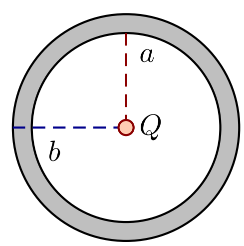
b) Determine the total charge on the outer surface of the conductor. Explain your answer.

c) Can you use your results to determine the charge density on the outer surface? Explain your answer.

d) Suppose that an additional point charge is placed anywhere outside the conductor. Which of the following stay the same and which change: electrostatic potential inside the cavity, charge density on the cavity, total charge on the outer surface, electric field beyond the conductor? Explain your answer.

e) Suppose that an additional cavity is created inside the conductor and another charge q is placed at the center of this. Which of the following stay the same and which change: electrostatic potential inside the original cavity, charge density on the original cavity, total charge on the outer surface? Explain your answer.

5 Charge in a conducting shell



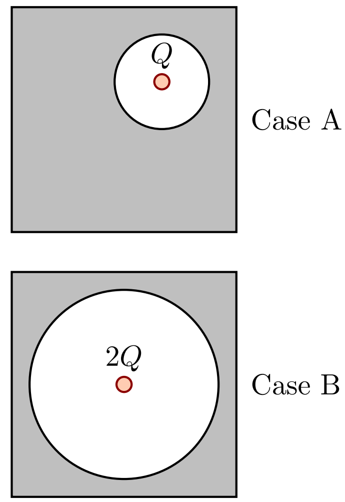
A point charge, Q is placed at the center of a thick conducting shell as illustrated.

a) Determine the electrostatic potential and electric field inside the shell ($r < a$).

b) Assuming that the shell was neutral before the point charge was placed at the center, determine the charge densities on both surfaces of the shell.

6 Cavity surface charge density

Two conductors each contain a spherical cavity. Each conductor is initially neutral. Subsequently a point particle is placed at the center of the cavity. The radius of the cavity in case A is R and that of case B is $2R$. The particle in the center of the cavity in case A has charge Q and that of case B has charge $2Q$. Let σ_A be the charge density on the inner surface of the cavity in case A and σ_B that of case B. Which of the following is true? Explain your answer.



i) $\sigma_A = \frac{1}{4}\sigma_B$.

ii) $\sigma_A = \frac{1}{2}\sigma_B$.

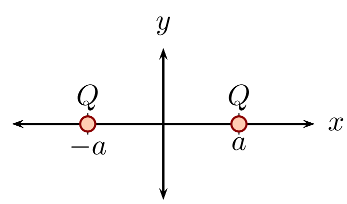
iii) $\sigma_A = \sigma_B$.

iv) $\sigma_A = 2\sigma_B$.

v) $\sigma_A = 4\sigma_B$.

7 Extrema of potentials due to point charges

Two identical point charges are situated as illustrated.



a) Determine an expression for the electrostatic potential $V(x, y)$ at all locations.

b) Use the potential to find locations where the net force on any charged particle is zero.

c) Describe whether these are local maxima or minima of the potential. Are they stable equilibria?

8 Laplace's equation in spherical coordinates

a) Consider a region in which there is no charge. Write Laplace's equation in terms of spherical coordinates. Solve this assuming that the potential only depends on r.

b) What type(s) of charge distribution could give this potential?

c) Is this potential applicable to the region beyond a charged spherical conductor? Explain your answer.

d) Is this potential applicable to the region beyond a charged rectangular conductor? Explain your answer.