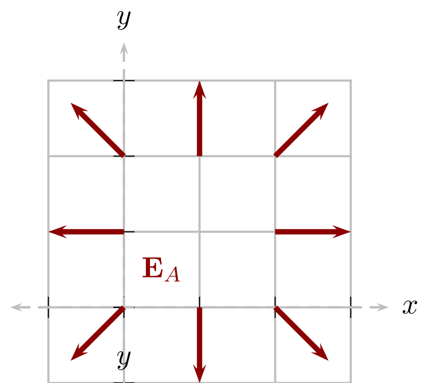
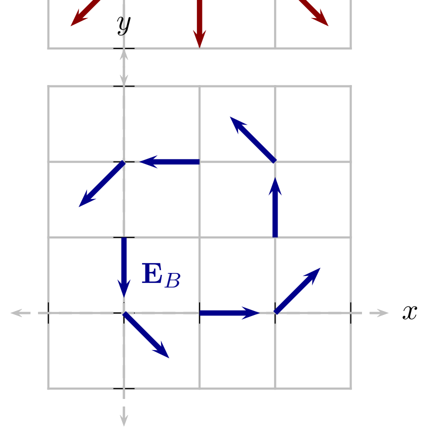
Electromagnetic Theory: Homework 11

Due: 25 September 2020

1 Electric fields and stationary source charges

Two proposed electric fields in the xy plane are illustrated. When the fields are extended to three dimensions along the z axis, they display the same pattern in each plane parallel to the xy plane. Describe whether each of these may be an electric field produced by stationary point charges. Explain your answers.





2 Possible electrostatic fields

a) Consider

$$\mathbf{E} = -k\left[\frac{1}{2}y^2\hat{\mathbf{x}} + \frac{1}{2}x^2\hat{\mathbf{y}}\right]$$

where k is a constant with appropriate units. Is this a possible electrostatic electric field? Explain your answer.

b) Consider

$$\mathbf{E} = -k\left[\frac{1}{r^2}\,\hat{\mathbf{r}} + 2\hat{\boldsymbol{\theta}} + \frac{1}{r}\,\hat{\boldsymbol{\phi}}\right]$$

where k is a constant with appropriate units. Is this a possible electrostatic electric field? Explain your answer.

3 Electric field and potential: sphere

A solid sphere of radius R carries charge whose density is as a function of position $\mathbf{r}'$ within the sphere is

$$\rho(\mathbf{r}') = \alpha r'$$

where r' is the distance from the center of the sphere and α is constant.

a) Determine an expression for the total charge, Q, in the sphere and use this to express α in terms of Q.

b) Use Gauss's Law to determine the electric field *anywhere* inside or outside the sphere; the answers must be expressed in terms of the total charge Q. Start with the field inside the sphere. Your solution *must contain in the following order:*

   i) A diagram of the charge distribution.

   ii) A simplification of the electric field using symmetry arguments.

   iii) An illustration and explanation of what Gaussian surface (the surface that appears in the surface integral) you are using. If this is the same as the surface of the charged sphere then all that you will calculate is the field on the surface of the sphere. That would be **incomplete and incorrect**. The illustration must indicate the enclosed charge.

   iv) Evaluation of the surface integral.

   v) Evaluation of the enclosed charge.

   You will have to repeat all but the first two parts when you determine the field outside the sphere.

c) Use the field that you calculated to determine the electrostatic potential at any location, setting $V = 0$ at any location infinitely far from the sphere.

d) Use the potential to compute the electric field. Check that your results are correct.

e) Check that Poisson's equation is correct for this distribution.

4 Electric field and potential: cylinder

An infinitely long hollow cylindrical pipe has an inner wall with radius a and outer wall with radius $b > a$. Between the walls, the pipe is charged uniformly with volume charge density ρ.

a) Use Gauss's Law to determine the electric field *anywhere*, i.e. inside the pipe's inner wall, within the material of the pipe and outside the pipe's outer wall . Your solution *must contain in the following order:*

   i) A diagram of the charge distribution.

   ii) A simplification of the electric field using symmetry arguments.

   iii) An illustration and explanation of what Gaussian surface (the surface that appears in the surface integral) you are using. If this is the same as the surface of the pipe then all that you will calculate is the field on the surface of the pipe. That would be **incomplete and incorrect**. The illustration must indicate the enclosed charge.

iv) Evaluation of the surface integral.

v) Evaluation of the enclosed charge.

b) Use the field that you calculated to determine the electrostatic potential difference between the center of the pipe and the inner wall.

c) Use the field that you calculated to determine the electrostatic potential difference between the center of the pipe and the outer wall.