Electromagnetic Theory: Homework 7

Due: 11 September 2020

1 Divergence and curl in spherical coordinates

Let

$$\mathbf{v} = 2\hat{\mathbf{r}} + 2\hat{\boldsymbol{\theta}}$$

in spherical coordinates.

a) Determine the divergence of $\mathbf{v}$.

b) Determine the curl of $\mathbf{v}$.

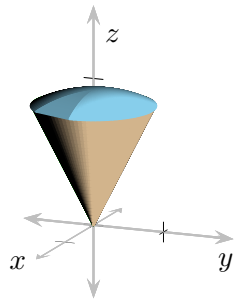
c) Is the sketch of $\mathbf{v}$ in the yz-plane consistent with your results?

2 Divergence theorem in spherical coordinates

Let

$$\mathbf{v} := r\cos\theta\,\hat{\mathbf{r}} - r\sin\theta\,\hat{\boldsymbol{\theta}} + r\cos\phi\,\hat{\boldsymbol{\phi}}.$$

Consider the conical section of a sphere of radius $a > 0$ as illustrated. The angle from the z axis to the conical section is $\pi/6$ and the top of the conical section is a portion of a sphere of radius a.



a) Determine $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ across the surface.

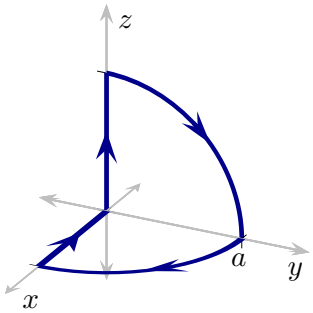
b) Verify the divergence theorem for this example.

3 Stokes' theorem in spherical coordinates

Let

$$\mathbf{v} := 2ar\hat{\boldsymbol{\theta}} + r^2\hat{\boldsymbol{\phi}}$$

where $a > 0$ is a constant.



a) Determine $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{l}$ along the illustrated curve.

b) Verify Stokes' theorem for this example.