Electromagnetic Theory: Homework 6

Due: 8 September 2020

1 Stokes' theorem

Let

$$\mathbf{v} = \frac{y}{2}\,\hat{\mathbf{x}} - \frac{1}{2x^3}\,\hat{\mathbf{y}}$$

and consider the path with straight line segments $(1,1,0) \rightarrow (2,1,0) \rightarrow (2,2,0) \rightarrow (1,2,0) \rightarrow (1,1,0)$. Verify that Stoke's theorem is true for this loop using the flat surface that it encloses.

2 Surface integrals for uniform fields

Consider an arbitrary vector field $\mathbf{v}$. The surface integral $\oint \boldsymbol{\nabla} \times \mathbf{v} \cdot d\mathbf{a}$ is computed over two surfaces: 1) a disk in the xy plane centered at the origin (normal points along $\hat{\mathbf{z}}$) and 2) a hemisphere whose base is the same as the disk and is above the base (normal is outward). How are the two surface integrals related? *Hint: Don't try to actually evaluate an integral. Think about Stokes' theorem.*

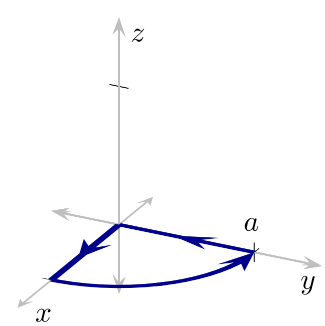
3 Cylindrical unit vectors

Consider two points in the $z = 0$ plane. Denote the point $(1,1,0)$ by P_1 and the point $(1,-1,0)$ by P_2. Is $\hat{\mathbf{s}}$ the same at P_1 as P_2? Is $\hat{\boldsymbol{\phi}}$ the same at P_1 as P_2? Explain your answers.

4 Stokes' theorem: cylindrical coordinates

Consider the vector field, in cylindrical coordinates,

$$\mathbf{v} = s\cos\phi\,\hat{\mathbf{s}} - s\sin\phi\,\hat{\boldsymbol{\phi}}$$

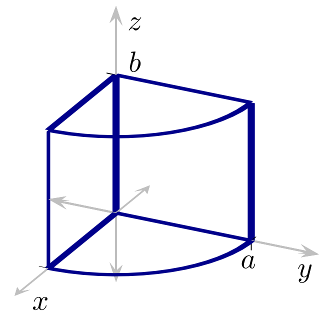


a) Sketch the vector field in the xy plane.

b) Determine the line integral along the illustrated path (the curved path is a section of a circle).

c) Verify that Stokes' theorem is true in this case.

5 Divergence theorem: cylindrical coordinates



Consider the vector field, in cylindrical coordinates,

$$\mathbf{v} = s\sin\phi\,\hat{\mathbf{s}} + s\cos\phi\,\hat{\boldsymbol{\phi}} + z\hat{\mathbf{z}}$$

and the illustrated surface which is a quarter cylinder of radius a and height b.

a) Determine the surface integral integral over the entire closed surface.

b) Verify that the divergence theorem is true in this case.