Electromagnetic Theory: Homework 5

Due: 4 September 2020

1 Fundamental theorem for gradients

Let

$$f(x, y, z) = x^2 y + x y^2 z.$$

a) Determine the gradient of f.
b) Determine the line integral of $\boldsymbol{\nabla} f$ over the line consisting of the straight line segments that run as follows: $(0,0,0) \to (1,0,0) \to (1,1,0) \to (1,1,1)$. (See Fig. 1.28)
c) Check that the fundamental theorem for gradients for this line integral.

2 Divergence theorem

Let

$$\mathbf{v} = 5xy^3\hat{\mathbf{x}} + 5yx^3\hat{\mathbf{y}}.$$

Consider the region enclosed by the rectangular box for which $0 \leqslant x \leqslant 2$, $0 \leqslant y \leqslant 2$, and $0 \leqslant z \leqslant 1$.

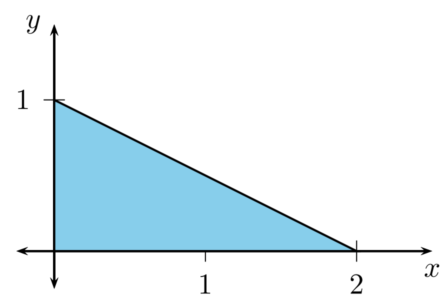
a) Determine $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ for the entire surface.
b) Determine $\int \boldsymbol{\nabla} \cdot \mathbf{v}\, \mathrm{d}\tau$ for the region and verify that the divergence theorem is satisfied.

3 Divergence theorem and a wedge

Let

$$\mathbf{v} = -2xz\hat{\mathbf{x}} + 3yz\hat{\mathbf{z}}.$$

and consider the surface consisting of the "wedge" region cross section in the xy plane is as illustrated and whose top is at $z = 1$ and bottom at $z = 0$.



a) Determine the surface integral for each of the five surfaces.
b) Determine $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ for the entire surface.
c) Determine $\int \boldsymbol{\nabla} \cdot \mathbf{v}\, \mathrm{d}\tau$ for the region and verify that the divergence theorem is satisfied.

4 Uniform vector fields

a) Let $\mathbf{v} = v\hat{\mathbf{y}}$, where $v > 0$ and consider the sphere centered at the origin. Is $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ positive, negative or zero for the spherical surface? Explain your answer.
b) Consider an arbitrary uniform vector field and *any* closed surface. Is $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ positive, negative or zero for the spherical surface? Explain your answer.