Electromagnetic Theory: Homework 4

Due: 1 September 2020

1 Charge density

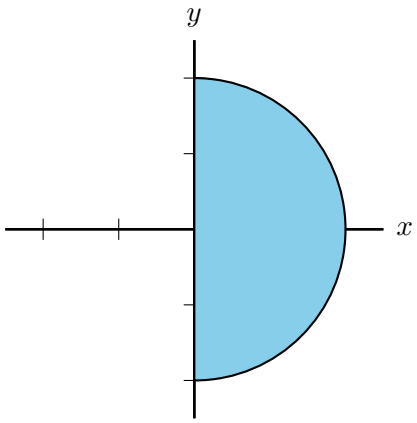
Charge is distributed with density

$$\rho = \frac{q}{a^4}\, x$$

where q is a constant with dimensions of charge.

a) Determine the total charge in the cube with sides of length a and corners at $(0,0,0)$, $(0,0,a)$, $(0,a,0)$, $(0,a,a)$, $(a,0,0)$, $(a,0,a)$, $(a,a,0)$, (a,a,a).

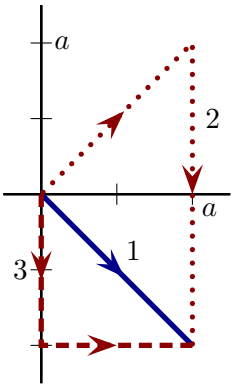
b) Determine $\int \rho \, d\tau$ for the region which consists of half of a cylinder that is parallel to the z axis, ranges from $z = 0$ to $z = a$, has radius a and whose base in the xy plane is as illustrated.



c) Explain without integrating what the total charge would be in a cylinder of the type described above but whose base is a full circle in the xy plane.

2 Line integrals in two dimensions: straight segments

Let $\mathbf{v} = y^2\hat{\mathbf{x}} - x^2\hat{\mathbf{y}}$. Three paths are indicated in the $z = 0$ plane.



a) Determine the line integral of $\mathbf{v}$ along line 1.

b) Determine the line integral of $\mathbf{v}$ along line 2: in the xy plane with straight line segments $(0,0) \to (a,a) \to (a,-a)$.

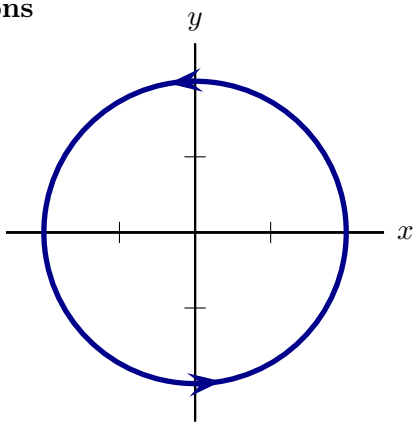
c) Determine the line integral of $\mathbf{v}$ along line 3: in the xy plane with straight line segments $(0,0) \to (0,-a) \to (a,-a)$.

3 Line integrals along a closed loop in two dimensions

Let

$$\mathbf{u} = y\hat{\mathbf{x}} - x\hat{\mathbf{y}} \quad \text{and}$$
$$\mathbf{v} = y\hat{\mathbf{y}}$$

Consider the line integral of each along the illustrated circular loop. Without actually computing the line integrals, describe whether the line integral of each vector field will be zero or not. Explain your answers. *Hint: sketch the vector fields.*



4 Line integrals in three dimensions

Let $\mathbf{v} = y\hat{\mathbf{x}} - x\hat{\mathbf{y}} + xy\hat{\mathbf{z}}$.

a) Determine the line integral of $\mathbf{v}$ along the line with straight segments $(0,0,0) \to (0,0,1) \to (0,1,1) \to (1,1,1)$.

b) Determine the line integral of $\mathbf{v}$ along the line with straight segments $(0,0,0) \to (0,0,1) \to (1,1,1)$.

c) Determine the line integral of $\mathbf{v}$ along the line with straight segments $(0,0,0) \to (1,0,0) \to (1,1,1)$.

d) Determine the line integral of $\mathbf{v}$ along the line with straight segment $(0,0,0) \to (1,1,1)$.

e) Does any line integral from $(0,0,0)$ to $(1,1,1)$ give the same result regardless of the path taken? Explain your answer.