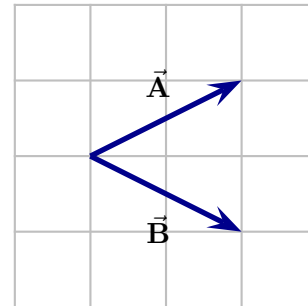


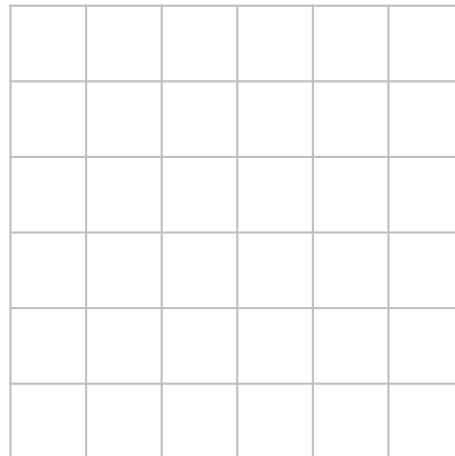
Laboratory 3: Vectors – Prelab

1 Adding and subtracting vectors

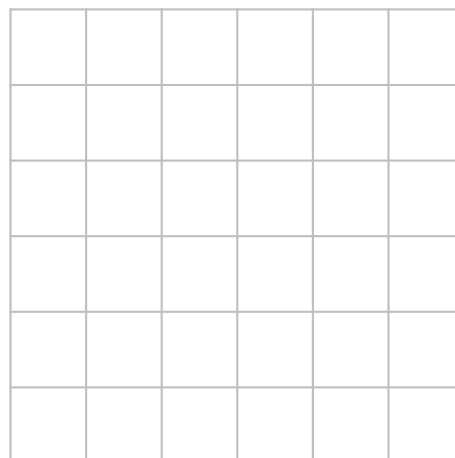
Two displacement vectors, \vec{A} and \vec{B} , are illustrated.



- a) Using the grid provided to the right, determine $\vec{C} = \vec{A} + \vec{B}$ by a graphical method. Use this to determine the magnitude C . Do the magnitudes satisfy $C = A + B$?



- b) Using the grid provided to the right, determine $\vec{D} = \vec{A} - \vec{B}$ by a graphical method. Use this to determine the magnitude D . Do the magnitudes satisfy $D = A - B$?



Laboratory 3: Vectors – Experiment/Tutorial

Vectors and vector algebra are used throughout classical physics. This laboratory exercise will illustrate vector algebra using physical and graphical methods and compare the results to those that can be obtained with purely mathematical constructions.

1 Walking or tracing displacement vectors

Displacement vectors describe the location of an object at a later instant relative to its location at an earlier instant. These vectors provide a useful conceptual illustration of vector addition: two displacements can be combined in succession to provide an overall displacement in the same way that two vectors, \vec{A} and \vec{B} , can be added, giving the sum $\vec{A} + \vec{B}$. This exercise illustrates such vector addition via tracing various displacements (*this is usually done by walking the displacements outside*).

Consider the following sequence of displacements:

$$\vec{A} = 10.0 \text{ cm due east and}$$

$$\vec{B} = 6.4 \text{ cm at an angle of } 20^\circ \text{ north of west}$$

and let $\vec{C} = \vec{A} + \vec{B}$. This is illustrated approximately in Fig. ??.

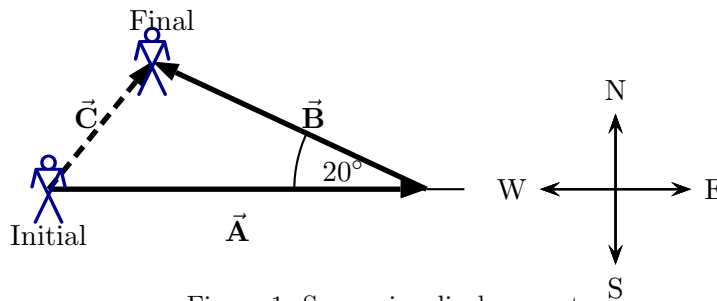


Figure 1: Successive displacements

The aim of this exercise will be to follow (by tracing) the two displacements successively and to measure the resulting overall displacement vector \vec{C} . The measured overall displacement will then be compared to a calculated overall displacement.

- Take a letter sheet piece of paper. Consider the bottom straight edge to be the “east” direction. Establish an initial location and mark this on the paper.
- Trace vector \vec{A} on the paper. The end of this vector will be the “intermediate location.”

- c) Starting at the intermediate location, trace the vector \vec{B} . The tip of this vector is the “final location.”
- d) Determine the overall displacement from your initial to final locations. This should consist of **both** a **distance** and a **direction** (e.g. angle measured from “east” toward “north” using the “line of sight” from the initial to the final location). The “line of sight” is illustrated for a different example in Fig. ??.

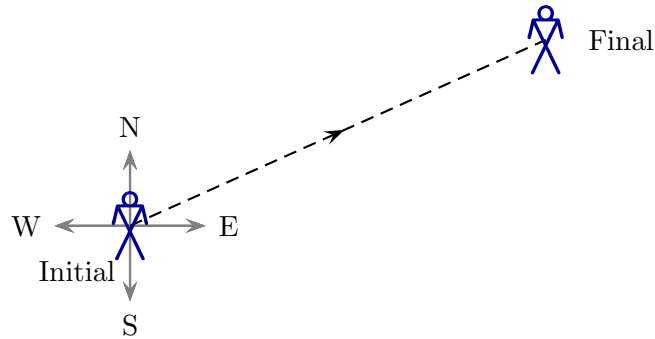


Figure 2: Overall displacement

Record the data for the displacement below.

- e) You will now use components to calculate the magnitude and direction of $\vec{C} = \vec{A} + \vec{B}$ **without using the results from the walking activity**. Let “east” correspond to the x axis and “north” the y axis when you do this. Determine the components of \vec{A} and \vec{B} and use them to find the components of \vec{C} . Then use these to sketch the vector \vec{C} and obtain its magnitude and angle. Table ?? might be useful.

Vector	x component	y component
\vec{A}		
\vec{B}		
\vec{C}		

Table 1: Displacement data.

- f) Do the magnitude and direction of the calculated \vec{C} agree with those of the measured displacement as determined via walking?
- g) Using the notation that A is the magnitude of \vec{A} , B is the magnitude of \vec{B} , etc, ..., is it true that $C = A + B$ in this exercise? Explain your answer.
- h) Is it true that $C^2 = A^2 + B^2$ in this exercise? Explain your answer.

2 Vector addition: graphical methods versus algebraic methods

A more accurate comparison of graphical methods and algebraic methods of adding vectors can be illustrated by drawing and adding vectors accurately on a sheet of paper. Consider the three vectors:

$\vec{A} = 8.0$ cm along the x axis,

$\vec{B} = 6.0$ cm at an angle of 45° counterclockwise from the x axis and

$\vec{C} = 12.0$ cm at an angle of 120° counterclockwise from the x axis.

Let $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. The aim of this exercise is to determine \vec{D} graphically and algebraically

- a) A graphical method for determining \vec{D} would first add $\vec{A} + \vec{B} + \vec{C}$ using the “head-to-tail” method. Draw these three vectors “head-to-tail” as accurately as possible (using a ruler and a protractor) on a separate sheet. Use this to draw \vec{D} .
- b) Determine the magnitude of \vec{D} graphically (i.e. by measuring from your drawing). Record this below.
- c) Draw the x and y components of \vec{D} on your diagram. Write the results below.
- d) Without using your drawing, calculate the components and magnitude of \vec{D} mathematically from the components of \vec{A} , \vec{B} , and \vec{C} using algebra and trigonometry (this is the “algebraic” method). How do these compare to result that you obtained graphically?

3 Vector subtraction: graphical methods versus algebraic methods

Vector subtraction is a variant of vector addition. Consider any two vectors \vec{v}_1 and \vec{v}_2 and define the difference between these as

$$\Delta\vec{v} := \vec{v}_2 - \vec{v}_1 \quad (1)$$

One way of constructing the difference is involves addition of a negative vector. Thus

$$\begin{aligned} \Delta\vec{v} &= \vec{v}_2 - \vec{v}_1 \\ &= \vec{v}_2 + (-\vec{v}_1) \end{aligned} \quad (2)$$

Thus one can perform the subtraction by adding the vectors \vec{v}_2 and $-\vec{v}_1$. Alternatively, the difference between the vectors must satisfy

$$\vec{v}_2 = \vec{v}_1 + \Delta\vec{v} \quad (3)$$

and, given the two vectors \vec{v}_1 and \vec{v}_2 , one can try to find $\Delta\vec{v}$ that satisfies Eq. (??).

For this exercise consider the vectors

$\vec{v}_1 = 5.0 \text{ cm}$ at an angle of 30° counterclockwise from the x axis and

$\vec{v}_2 = 5.0 \text{ cm}$ at an angle of 150° counterclockwise from the x axis.

- a) On a separate sheet of paper, draw a set of x and y axes and draw \vec{v}_1 and \vec{v}_2 as accurately as possible.
- b) The vectors \vec{v}_1 and \vec{v}_2 have the same magnitude. Does this mean that $\vec{v}_2 - \vec{v}_1 = 0$? Explain your answer.

- c) Use the scheme of Eq (??) to draw $\Delta\vec{v}$ as accurately as possible on the same sheet of paper. Use your drawing to describe the direction of $\Delta\vec{v}$ and to determine its magnitude. Record these below.

- d) Calculate the components of $\Delta\vec{v}$ using trigonometry and algebra. Use the result to describe the magnitude and direction of $\Delta\vec{v}$. How do these calculated results compare to your measured results?

- e) Use the “head-to-tail” method of addition and accurate drawings of \vec{v}_1 and $\Delta\vec{v}$ to construct $\vec{v}_1 + \Delta\vec{v}$ graphically. How does the resulting vector compare to \vec{v}_2 ? What does this imply (correct, incorrect, maybe correct) about Eq. (??)?

4 Unit vectors and vector components

Any vector \vec{A} can be expressed in terms of unit vectors as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} \quad (4)$$

where A_x and A_y are the components of \vec{A} . This exercise will illustrate why this is possible and how this decomposition can be used to add vectors. Consider the two vectors:

$\vec{A} = 10$ cm at an angle of 53° counterclockwise from the x axis and

$\vec{B} = 5.0$ cm at an angle of 37° clockwise from the x axis.

- a) On a separate sheet of paper draw a set of x and y axes. Draw the vector \vec{A} as accurately as possible (using a ruler and a protractor).

- b) Show, by drawing, how to construct \vec{A} as accurately as possible by repeatedly adding \hat{i} (a unit vector with length 1 cm along the x axis) to itself followed by repeatedly adding \hat{j} (a unit vector with length 1 cm along the y axis).

An example for a different vector \vec{D} is illustrated in Fig. ???. Constructing \vec{D} required 4 copies of \hat{i} and 2 copies of \hat{j} . Thus $\vec{D} = 4\hat{i} + 2\hat{j}$.

How many copies of each of these (reversed versions of the unit vectors count negatively, e.g. -1 copy) were required to construct \vec{A} ?

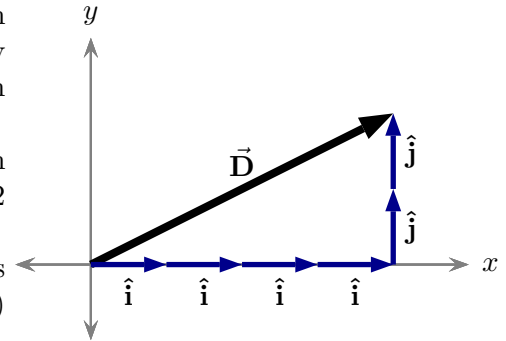


Figure 3: Unit vector decomposition

Use the answer to this question to write

$$\vec{A} = (??)\hat{i} + (??)\hat{j}$$

where the parentheses should be replaced by numbers.

- c) Calculate the components of \vec{A} using trigonometry and algebra. Substitute these into Eq. (??). Does the resulting expression agree with that which you obtained in part (b)?

d) Repeat parts (a) to (c) for $\vec{\mathbf{B}}$.

e) Construct $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ graphically by drawing a succession of unit vectors, first those for $\vec{\mathbf{A}}$ and then for $\vec{\mathbf{B}}$. Verify that the result would be the same by first following all the copies of $\hat{\mathbf{i}}$ for both vectors and then all the copies of $\hat{\mathbf{j}}$ for both vectors (you will have to be careful about directions) Write the resulting vector in the form

$$\vec{\mathbf{C}} = (??)\hat{\mathbf{i}} + (??)\hat{\mathbf{j}}.$$

How does this compare to what you would calculate using the components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$?



