Laboratory 13: Radioactive Dice – Prelab

Radioactive materials decay in a statistically predictable way. For any given material the half life is the amount of time that it takes for half of the radioactive atoms to decay.

1 Radioactive material

A sample of a radioactive material contains 160000 atoms at an initial instant. The half life of this material is $15\,\mathrm{hr}$. Determine the number of atoms that remain $45\,\mathrm{hr}$ after the initial instant.

2 Exponential decay

Two samples of a radioactive material each contain N_0 atoms at time $t = 0$. At later times the number of undecayed atoms in sample A is

$$N_A = N_0 e^{-20t}$$

and for sample B,

$$N_B = N_0 e^{-10t}.$$

Which sample decays more rapidly? Explain your answer.

Laboratory 13: Radioactive Dice – Activity

Some atoms have the remarkable property of being unstable, and decay into other elements by emitting different particles associated with different decay processes. Alpha, beta, and gamma decay lead to very different radioactive particles being emitted by the radioactive element, but in alpha and beta decay the decay process transforms the decaying element into another element. In addition, because the radioactivity is dependent on the number of radioactive atoms, and the probability of a radioactive event occurring, the mathematical description of the population of radioactive atoms is the same. The purpose of this lab is to model radioactive decay using ordinary six-sided dice.

Within a population of radioactive particles N, the number of radioactive particles that actually decay ΔN in a time interval Δt is given by

$$-\frac{\Delta N}{\Delta t} = \lambda N \tag{1}$$

where λ is known as the decay constant, related to the probability of the particle to decay. Performing some calculus on Eq. (1) gives an exponential relationship between the number of particles remaining in a population of radioactive particles N and the original population N_0, given by

$$N = N_0 \, e^{-\lambda t} \tag{2}$$

where t is the elapsed time from the original population.

1 Theory and Experimental Design

In this experiment, the class will be rolling a large number of dice to simulate a large population of radioactive atoms. If the result of any die is a 6, it "decays" and is removed from the population.

a) What is the decay constant of any individual die? Suppose that you throw the dice once every second. The decay constant can be determined from Eq. (2) by substituting in the fraction of dice that you expect to remain after a single throw of the dice, N/N_0. Use this to determine the decay constant.

b) Often when discussing radioactive elements, the term half-life appears. One half-life is defined as the time necessary for a population of radioactive elements to decay to one half of the original population. Determine the half-life (in units of number of throws of remaining dice) for the radioactive dice.

Here are some useful mathematical relationships involving the natural logarithm:

$$\ln e^a = a \tag{3}$$

$$\ln a^{-1} = -\ln a \tag{4}$$

$$\ln (ab) = \ln a + \ln b \tag{5}$$

c) You will be counting the number of remaining, undecayed, dice after every throw of the remaining dice and removing any "decayed" dice. Predict and sketch what you expect a graph of the remaining, undecayed, dice versus time (in units of throws of the dice) to look like. Is this graph linear?

d) Because a linear fit to data is desirable for determining physical quantities from measurements and comparing to theoretical predictions, linearize Eq. (2) such that you have a linear relationship in the form $y = mx + b$. You will likely need to use the aforementioned mathematical relationships involving the natural logarithm. Once you have the linearized equation, answer the following questions:

   i) What will you plot on the y-axis of your linearized graph?

   ii) What will you plot on the x-axis of your linearized graph?

   iii) What should the slope of your graph be?

   iv) What should the y-intercept of your graph be? **You won't be able to plug in an actual number until you begin the experiment with the entire class.**

2 Experiment – Radioactive Dice

Because statistical experiments such as radioactivity rely upon large populations, you will be rolling dice along with the entire class and announcing the number of dice from your individual population that decay. The instructor will keep track and record the entire population in the classroom on the whiteboard. To ease the amount of counting, the instructor will ask for the number of decayed dice that you have removed, instead of the remaining, undecayed dice.

a) Open `Excel` and create a new spreadsheet having 3 columns identified as: `roll number`, `N`, `dN`. `Roll number` represents time and identifies how many times the remaining population of dice have been rolled. `N` represents the total population of remaining, undecayed dice within the entire classroom. `dN` represents the total number of dice that decayed within the entire classroom on the respective `roll number`.

b) Wait for the entire class to be ready for the experiment. Count your entire population of dice and inform the instructor of that number. The instructor will tally the total population of dice and write it on the whiteboard.

c) Along with the rest of the class, roll all remaining, undecayed dice (this will be all of the dice on the first throw). Count up and remove any dice that rolled a 6. Inform the instructor who will then tally up the decayed dice within the classroom. Record the `Roll number`, N, and `dN` into your `Excel` worksheet. Remember that these numbers should be totals for the entire classroom. Repeat this process for 20 throws of the dice! Make sure to wait for the entire class before rolling your dice.

d) Use `Excel` to create a graph of N versus `roll number`. Does the graph agree with your prediction from part 1.c)? Print out the graph and attach it to your work.

e) By examining the graph of N versus `roll number`, estimate the half-life of the radioactive dice? Does this agree with your prediction from part 1.b)?

f) Use `Excel` to perform any necessary calculations so that you can create the linearized graph you identified in part 1.d). Is your graph actually linear? Have `Excel` fit a trendline to the data and record the slope and y-intercept of this. Print out the graph and attach it to your work.

g) Calculate percent differences for both your measured decay constant λ and y-intercept by comparing with the theoretical values using

$$\frac{|E - K|}{K} \times 100\% \qquad (6)$$

where E is the experimental, measured value determined from the linear regression and K is the known, theoretical value.

3 Conclusions

a) Comment on the agreement of your measured decay constant with the predicted dice constant. Because common game dice use pips where part of the plastic is drilled out of the die, the faces have very slight differing weights.

b) Carbon-14 is produced when cosmic rays interact with the upper atmosphere and because of the carbon cycle, all living plants and animals have a relatively stable carbon-14 to carbon-12 ratio. Once an organism dies, the carbon-14 within begins to decay. The known half-life of carbon-14 is 5,730 years. If an unearthed fossil contains 15% of the expected carbon-14 in a living organism, how long ago did that creature die?