Phys 112 2019

Lecture 39

Tues: Regular lecture

Weds: Discussion / quiz
Supp 131, 132, 133, 134
Ch 29 Q 8
Ch 29 Prob 26, 34, 35

Atomic models: hydrogen atom

A classical physics model of a hydrogen atom consists of an electron orbiting a (nearly) stationary proton. Classical physics allows for this and describes:

1) the acceleration of the electron
2) the relationship between orbital speed and radius of orbit of the electron.
3) the energy of the electron
4) the electromagnetic waves radiated by the accelerating electron.

Classical electromagnetism predicts the rate at which the atom loses radiation to the electromagnetic wave that it produces. The result is that a hydrogen atom should collapse in about 10^{-11}s.

Demo PhET Hydrogen atom model
- classical solar system

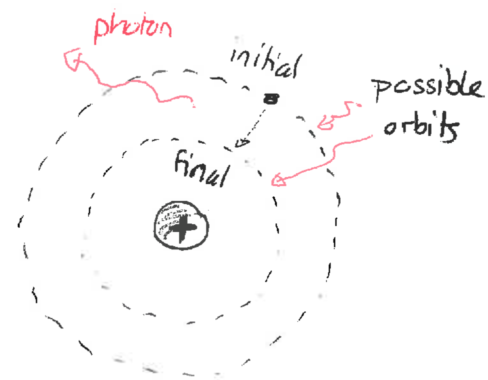
The classical model clearly cannot work and we need a model which
1) predicts the discrete spectrum of an atom
2) allows for the atom to remain stable

Bohr model

The first successful model to predict the atom spectra was provided by Niels Bohr in 1913. The model describes the hydrogen atom and consists of:

1) an electron can orbit in certain particular circular orbits without producing electromagnetic waves and losing energy

2) the electron can absorb or emit electromagnetic radiation if it jumps from one stable orbit to another. The wavelength of the emitted radiation is given by:



$$|\Delta E_{atom}| = E_{photon} = \frac{hc}{\lambda}$$

$$\Rightarrow \frac{hc}{\lambda} = |E_{atom\,f} - E_{atom\,i}|$$

Demo: PhET Hydrogen Atom Model
– Bohr model

3) the frequency of the light emitted is not the frequency of either the initial or final orbit.

Quiz 1 50% - 70%

These could describe the qualitative aspects of the spectrum of the hydrogen atom. The precise rules for the energy levels required one more assumption

4) a given orbit has energy E related to the frequency of orbit by
$E = \frac{1}{2} n f$

where $n = 1, 2, 3, \ldots$ is an integer.

The rest of the model follows a classical physics analysis and results in:

The possible values of energy for the hydrogen atom are:

$$E_n = -\frac{2\pi^2 m k^2 e^4}{h^2} \frac{1}{n^2}$$

where
$k =$ Coulomb constant
$m =$ mass of electron
$e =$ electron charge
$h =$ Planck's constant
$n = 1, 2, 3 \ldots$

A more convenient expression is:

$$E_n = E_1/n^2 \qquad n = 1, 2, 3, \ldots$$

where

$$E_1 = -\frac{2\pi^2 m k^2 e^4}{h^2} = -2.17 \times 10^{-18}\,J = -13.6\,eV$$

This gives a discrete set of possible energies. The lowest energy level is called the ground state ($n=1$)

$n=3$ ——— $E_3 = -1.51\,eV$
$n=2$ ——— $E_2 = -3.4\,eV$

$n=1$ ——— $E_1 = -13.6\,eV$

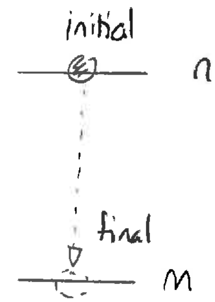
Quiz 2 60% →

Warm Up 1

Warm Up 2

Hydrogen atom spectrum

We can check whether this model correctly predicts the hydrogen atom spectrum. We consider a hydrogen atom that undergoes a jump from the level n to the level m.



Then

$$\Delta E_{atom} = E_n - E_m$$

$$= \frac{E_1}{n^2} - \frac{E_1}{m^2}$$

$$= E_1 \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

But $\Delta E_{atom} = \frac{hc}{\lambda}$ implies

$$\frac{hc}{\lambda} = E_1 \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \Rightarrow \frac{hc}{E_1 \left(\frac{1}{n^2} - \frac{1}{m^2} \right)} = \lambda$$

$$\Rightarrow \lambda = \frac{hc}{E_1} \frac{1}{\left(1/n^2 - 1/m^2 \right)}$$

Then $$hc/E_1 = \frac{6.6 \times 10^{-34}\, \cancel{J} \cdot \cancel{s} \times 3.0 \times 10^{8}\, m/\cancel{s}}{-2.17 \times 10^{-18}\, \cancel{J}} = -91.1nm$$

So

$$\lambda = \frac{-91.1nm}{1/n^2 - 1/m^2} = \frac{91.1nm}{1/m^2 - 1/n^2}$$

This is the Balmer formula.

Quantum theory model of the hydrogen atom

There were various deficiencies inherent in the Bohr model. In the subsequent 10-15 years a new model was developed based on quantum theory. This has the ingredients:

1) there exist states of the electron in which it does not radiate energy
2) these states are describe in terms of a wave which must satisfy the Schrödinger equation
3) there is no definite circular orbit or radial distance from the nucleus. Even for one particular energy state there are multiple radial distances possible. The wave gives a method for finding the probability with which the electron can be found at various locations

Demo: PhET H atom model – Schrödinger

The results of this model are:

1) the possible energies are

$$E_n = -13.6\,\text{eV}\,\frac{1}{n^2} \qquad n = 1, 2, \ldots$$

2) there are usually multiple states for each value of n
3) location probabilities have the form