Electromagnetic Theory: Homework 24

Due: 5 December 2019

1 Induced field due to a current in an infinitely long straight wire

An infinitely long straight wire lies along the z axis and carries a current $I(t)$. We aim to determine the electric field that this induces beyond the wire. *(Note: this ignores certain subtle EM issues: the impossibility of having the current change in the same way instantaneously at all points along the wire and the finite speed with which electromagnetic signals can propagate beyond the wire. It will give approximate solutions.)*

a) Determine a set of differential equations for the components of the induced electric field.

b) Suppose that the induced electric field also has zero divergence. Determine the resulting differential equation that the field satisfies.

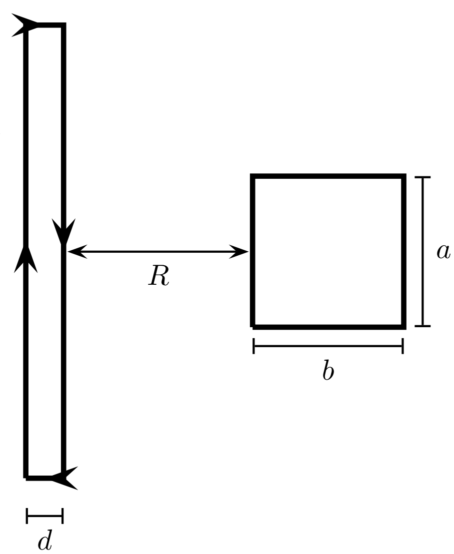
c) By symmetry, the induced electric field cannot depend on ϕ and z (in cylindrical coordinates). Use this to simplify all the differential equations.

d) Solve the differential equations to obtain a general expression for the electric field in terms of the current.

2 Griffiths, *Introduction to Electrodynamics, 4ed*, 7.15, page 320.

3 Mutual inductance

Two rectangular loops are situated as illustrated. The loop on the left is very long compared (you can assume the the vertical length is infinite) to its width and to the dimensions of the loop on the right. The situation is as illustrated, with the vertical length of the loop on the left drawn at a greatly reduced scale.

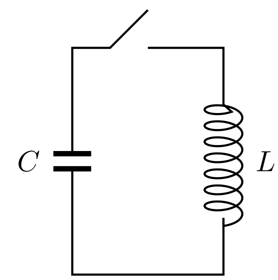


a) Suppose that current I flows through the loop at the left. Determine the flux through the loop at the right.

b) Determine the mutual inductance of this arrangement.

4 LC circuit

A capacitor with capacitance C, an inductor with inductance L and a switch are connected in series. While the switch is open, the capacitor is charged by connecting it to a battery with potential difference ΔV. The battery is then disconnected and after that the switch is closed.



a) Determine a differential equation satisfied by the charge on the capacitor after the switch is closed.

b) Solve the differential equation for the charge on the capacitor with $t = 0$ being the time at which the switch is closed. Express the solution in terms of ΔV.

c) After how long does the charge on the capacitor return to its value at the moment when the switch was closed?

5 Energy associated with a spinning uniformly charged spherical shell

A spherical shell of radius R contains a surface charge of density σ. The shell rotates with constant angular velocity ω about the z-axis. In Example 5.11 it was shown that the magnetic vector potential for this is

$$\mathbf{A} = \begin{cases} \dfrac{\mu_0 R\omega\sigma}{3}\, r\sin\theta\,\hat{\boldsymbol{\phi}} & \text{if } r \leqslant R \text{ and} \\ \dfrac{\mu_0 R^4\omega\sigma}{3}\,\dfrac{\sin\theta}{r^2}\,\hat{\boldsymbol{\phi}} & \text{if } r \geqslant R. \end{cases}$$

a) Show that the magnetic field is

$$\mathbf{B} = \begin{cases} \dfrac{2\mu_0 R\omega\sigma}{3}\,\hat{\mathbf{z}} & \text{if } r \leqslant R \text{ and} \\ \dfrac{\mu_0 R^4\omega\sigma}{3r^3}\left[2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right] & \text{if } r \geqslant R. \end{cases}$$

b) Determine the total energy stored in this magnetic field. You are encouraged to use MAPLE or an equivalent program to calculate the integrals.