Electromagnetic Theory: Homework 21

Due: 12 November 2019

1 Magnetic field produced by a rotating ring of charge

A circular ring has radius R and carries charge Q that is uniformly distributed. The ring lies in the xy plane and rotates with angular velocity ω about an axis along the z axis.

a) Determine the magnetic dipole moment of the ring.

b) Determine the dipole magnetic field produced by the ring.

c) The angular momentum of this ring is $\mathbf{L} = MR^2\omega\mathbf{z}$ where M is the mass of the ring. Determine a relationship between $\mathbf{L}$ and the magnetic dipole moment in terms of Q and M.

2 Square loop of current

A square lying in the xy plane has sides with length L and carries current I.

a) Determine the magnetic dipole moment of the square.

b) Determine the dipole magnetic vector potential of the square and use this to approximate the magnetic field produced by the current.

c) The text shows how to use the Biot-Savart law to determine the magnetic field produced by a segment of straight current. Use this to determine an exact expression for the field along the z axis.

d) Show that for $z \gg L$ the dipole approximation for the field approaches the exact expression for the field.

e) By what fraction does the dipole approximation for the field at the center of the square differ from the exact expression at the center of the square?

3 Magnetic dipole due to a rotating disk

A disk of radius R carries a surface charge with uniform density σ. The disk rotates with constant angular velocity ω.

a) Verify by explicit calculation that the magnetic monopole moment of this is zero.

b) One way to determine the magnetic dipole moment is to consider the disk as a collection of circular rings of current. Use the result for the magnetic dipole moment for a ring of current to determine the magnetic dipole moment of the disk.

c) Another way to determine the dipole moment is

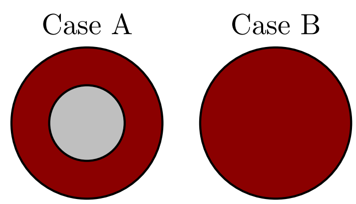
$$\mathbf{m} = \frac{1}{2}\int \mathbf{r}' \times \mathbf{K}(\mathbf{r}')\mathrm{d}a'.$$

Use this to determine the magnetic dipole moment of the rotating disk.

4 Griffiths, *Introduction to Electrodynamics, 4ed*, 5.34, page 255.

5 Rotating disks: different charge distributions

Two circular disks have the same radius and uniformly distributed mass and each carry the same total charge. In case A the charge is uniformly distributed on the outer section of the disk (from halfway toward the edge up to the edge). In case B it is uniformly distributed throughout the disk. Both disks rotate with the same angular velocity about an axis perpendicular to the page.



a) How does the magnetic dipole moment for A compare to that for B? Explain your answer.

b) How does the angular momentum for A compare to that for B? What does this imply for the ratio of angular momentum to dipole moment for the two cases? Explain your answer.