Electromagnetic Theory: Homework 20

Due: 8 November 2019

1 Magnetic Vector Potentials

For each of the following potentials, given in spherical coordinates, determine the associated magnetic field and the current density that produces these.

a) $\mathbf{A} = k\hat{\boldsymbol{\phi}}$ where k is constant.

b) $\mathbf{A} = k\hat{\boldsymbol{\theta}}$ where k is constant.

2 Vector potential for an infinite sheet of current

A uniform surface current flowing in the xy plane, described by surface current $\hat{\mathbf{K}} = K\hat{\mathbf{x}}$ generates a magnetic field

$$\mathbf{B} = \begin{cases} \dfrac{\mu_0 K}{2}\,\hat{\mathbf{y}} & \text{for } z > 0 \\ -\dfrac{\mu_0 K}{2}\,\hat{\mathbf{y}} & \text{for } z < 0 \end{cases}$$

a) Is it possible to find a magnetic vector potential of the form $\mathbf{A} = A\hat{\mathbf{y}}$ for this field? Explain your answer.

b) Find a vector potential that satisfies $A_x = A_y = 0$. Denote this $\mathbf{A}_1$ and sketch it in the xz plane.

c) Find a vector potential that satisfies $A_z = A_y = 0$. Denote this $\mathbf{A}_2$ and sketch it in the xz plane.

d) Show that

$$\mathbf{A} = \frac{1}{2}\,(\mathbf{A}_1 + \mathbf{A}_2)$$

generates the same magnetic vector field. Sketch this in the xz plane.

3 Choice of vector potential

Consider an infinite cylinder of radius R that carries current that flows down the length of the cylinder with uniform density. The magnetic field that this produces is

$$\mathbf{B} = \begin{cases} \dfrac{\mu_0 I}{2\pi s}\,\hat{\boldsymbol{\phi}} & \text{for } s > R \\ \dfrac{\mu_0 I s}{2\pi R^2}\,\hat{\boldsymbol{\phi}} & \text{for } s < R \end{cases}$$

One possibility for the magnetic vector potential is

$$\mathbf{A} = \begin{cases} -\dfrac{\mu_0 I}{2\pi} \ln(s)\, \hat{\mathbf{z}} & \text{for } s > R \\[2ex] -\dfrac{\mu_0 I s^2}{4\pi R^2}\, \hat{\mathbf{z}} & \text{for } s < R \end{cases}$$

a) Check whether $\mathbf{A}$ has zero divergence.

b) Check whether $\mathbf{A}$ satisfies

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}.$$

If not explain why not.

c) Let

$$\mathbf{A}' := \mathbf{A} + z^2 \hat{\mathbf{z}}.$$

Verify that this generates the magnetic field produced by this current distribution. Is the divergence of $\mathbf{A}'$ zero?

4 Magnetic field produced by rotating charged spheres

The text calculates the magnetic vector potential produced by a spinning charged shell.

a) Determine the magnetic field outside the shell.

b) Determine an expression for the magnitude of the magnetic field outside the sphere, showing that it is proportional to $\sqrt{3\cos^2\theta + 1}$.

c) Consider a rotating solid sphere with uniform charge density ρ. Use the result for a shell to determine the magnetic vector potential *outside* the sphere. *Hint: You will have to break the sphere into suitable sections and add the contributions from each section.*

d) Describe *how* you would have to modify your calculation to determine the magnetic vector potential inside the sphere.