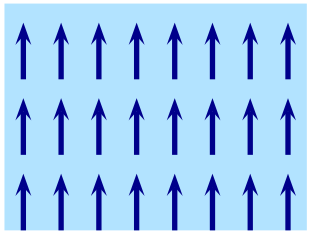
Electromagnetic Theory: Homework 19

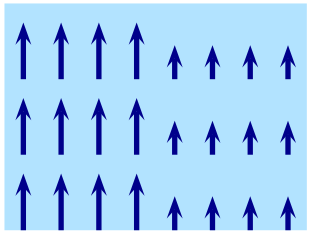
Due: 5 November 2019

1 Source currents for magnetic fields

a) A region contains a uniform magnetic field as illustrated. Are there any source currents in this region? If so, describe their direction. Explain your answer.



b) A region contains a magnetic field as illustrated. Within the left half the magnetic field is uniform and within the right half it is also uniform but has a different magnitude. At a thin boundary between the regions the magnitude decreases rapidly over space. Are there any source currents in this region? If so, describe their direction. Explain your answer.

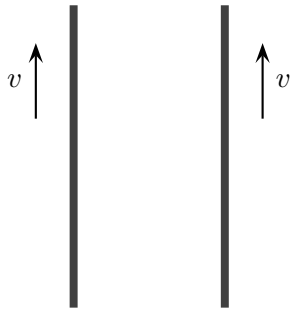


2 Magnetic field produced by a uniform sheet

An infinite sheet lies in the xy plane carries charge that is uniformly distributed with density σ. The sheet is dragged along the y axis with speed v. Determine the magnetic field produced by the sheet and verify that the magnitude of the field is $B = \mu_0 \sigma v/2$.

3 Forces between infinitely long moving charged wires

Two infinitely long wires each hold charge with linear charge density λ. They are parallel to each other and separated by distance d. The wires are dragged with constant speeds v along their axes as illustrated.



a) Determine the electric field produced by the wire on the left at the location of the wire on the right and use this to determine the electric force per unit length on the wire on the right.

b) Determine the magnetic field produced by the wire on the left at the location of the wire on the right and use this to determine the magnetic force per unit length on the wire on the right.

c) Determine the ratio of the electric force per unit length to the magnetic force per unit length. Note that the speed of light in a vacuum is $c = 1/\sqrt{\epsilon_0 \mu_0}$. With what velocity must the wires be dragged so that the forces are equal?

d) Suppose that the charge densities in the two wires were different to each other. Would this affect the ratio of the electric force per unit length to the magnetic force per unit length. Explain your answer.

4 Magnetic field produced by a beam of charged particles

A beam of positively charged particles moves along the positive z axis with constant velocity $\mathbf{v} = v\,\hat{\mathbf{z}}$. The beam has a circular cross-section with radius a and the charge density is uniform with density ρ. Consider the extreme case where the beam is infinitely long.

a) Determine the electric field at any point inside or outside the beam.

b) Determine the magnetic field at any point inside or outside the beam.

c) At any location, determine the ratio of the magnitudes of the fields E/B. Simplify this using the fact that the speed of light in a vacuum is $c = 1/\sqrt{\epsilon_0 \mu_0}$.

d) Suppose that a single charge particle is moves with velocity $\mathbf{u} = u\,\hat{\mathbf{z}}$ outside the beam. Determine expressions for the electric and magnetic forces exerted on this charge. Which is larger? When are they equal in magnitude?