Electromagnetic Theory: Homework 15

Due: 18 October 2019

1 Multipole expansion for three dimensional distributions

a) Inside a sphere of radius R the charge density is

$$\rho(\mathbf{r}') = \frac{3q}{4\pi R^3}.$$

Sketch this qualitatively and determine the dipole and monopole moments of this. Use these to obtain an approximate expression for the potential produced by this distribution.

b) Inside a sphere of radius R the charge density is

$$\rho(\mathbf{r}') = \frac{3\alpha}{4\pi R^3}\ \sin\phi'.$$

Sketch this qualitatively and determine the dipole and monopole moments of this. Use these to obtain an approximate expression for the potential produced by this distribution.

2 Multipole expansion for one dimensional distributions

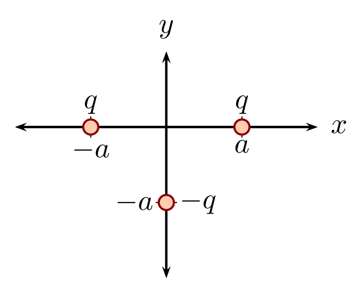
A one-dimensional circular loop of wire has radius R and lies in the xy plane. Charge is distributed along the loop (in spherical coordinates) according to

$$\lambda(\mathbf{r}') = \lambda \sin\left(\frac{\phi'}{2}\right)$$

a) Determine the monopole and dipole terms in the potential that this produces.

b) *This part is a **bonus** problem, worth a total of 10 bonus points.* Determine the quadrupole term in the expansion for the potential.

3 Monopole expansion for point charges



Several point charges are situated as illustrated. Each is distance a from the origin.

a) Determine an expression for the monopole and dipole moments of this distribution.

b) Use these moments to determine an approximate expression for the electrostatic potential produced by the distribution.

4 Force exerted by a dipole

A dipole is located at the origin and has dipole moment $\mathbf{p} = p\,\hat{\mathbf{x}}$.

a) Determine an expression, in spherical coordinates, for the electrostatic potential produced by the dipole.

b) Determine an expression, in spherical coordinates, for the electric field produced by the dipole.

c) A point particle with charge q is located at $(a, 0, 0)$ (in Cartesian coordinates). Determine the force exerted by the dipole on the charge.

d) A point particle with charge q and mass m is held at rest at $(a, 0, 0)$ (in Cartesian coordinates). The particle is released. Determine an expression for its speed when it is infinitely far from the origin.

5 Electric dipole potential and field

A spherical shell contains charge which is distributed according to (in spherical coordinates)

$$\rho(\mathbf{r}') = \alpha\cos(\theta')$$

where $\alpha > 0$ is a constant.

a) Which of the following best represents the dipole moment? In the following $p > 0$.

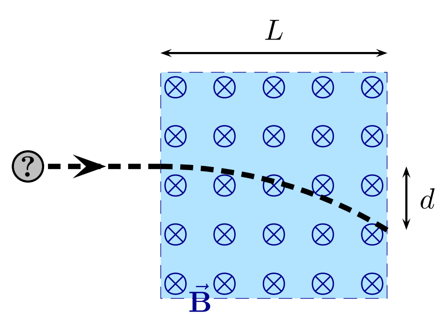
   i) $\mathbf{p} = p\hat{\mathbf{z}}$
   ii) $\mathbf{p} = -p\hat{\mathbf{z}}$
   iii) $\mathbf{p} = p\hat{\mathbf{x}}$
   iv) $\mathbf{p} = -p\hat{\mathbf{x}}$
   v) $\mathbf{p} = p\hat{\mathbf{y}}$
   vi) $\mathbf{p} = -p\hat{\mathbf{y}}$

b) Are there any locations where the dipole potential satisfies $V_{\text{dipole}} = 0$? If so, describe them.

c) Are there any locations where the dipole electric field satisfies $\mathbf{E}_{\text{dipole}} = 0$? If so, describe them.

6 Circular motion of a particle in a constant field

A particle is fired into a region with constant magnetic field pointing into the page. The trajectory of the particle is as illustrated. The width of the field region is L and the particle is deflected by amount d at the point that it leaves the field.



a) Is the particle positive or negative? Explain your answer.

b) Determine an expression for the radius of orbit of the particle in terms of L and d.

c) Determine an expression for the momentum of the particle in terms of L, d, B and the charge q.

7 Particle motion in a constant magnetic field

A charged particle of mass m and charge Q is in a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$. At $t = 0$ the particle is located at $x = x_0 > 0$ and $y = z = 0$. The velocity of the particle at $t = 0$ has components $v_x(0) = v_z(0) = 0$ and $v_y(0) = v$ where v is the speed of the particle.

a) Determine expressions for $x(t), y(t)$ and $z(t)$ at all later times.

b) Show that particle moves in a circle in the xy plane. Determine an expression for the location of the center of the circle and for the radius of the circle. *Hint: any circular trajectory can be expressed as*

$$[x(t) - a_1]^2 + [y(t) - a_2]^2 + [z(t) - a_3]^2 = R^2$$

where R is the radius of the circle and (a_1, a_2, a_3) its center.

c) Show that the radius of orbit satisfies $R = mv/QB$.