

## Electromagnetic Theory: Homework 12

Due: 1 October 2019

### 1 Potential produced by a uniformly charged wire

A wire with length  $L$  carries a charge with uniform density  $\lambda$ .

- Determine an expression for the electric potential at any point  $P$  along the axis of the wire but beyond the wire.
- Describe whether you can use your result to compute the electric field at point  $P$ . Can you compute any component of the electric field?



### 2 Potential produced by a hemisphere

A hollow surface consists of a hemisphere with radius  $R$ . The surface holds a uniform positive charge density  $\sigma$ .

- Show that the magnitude of the potential difference between the pole of the hemisphere and the point at the center of its (circular) base is

$$\frac{R\sigma}{2\epsilon_0} (\sqrt{2} - 1).$$

- Suppose that a positive charge is released from rest inside the hemisphere at the north pole. If the charge is positive which way will it move? If the charge is negative, which way will it move?

### 3 Work and kinetic energy in electrostatics

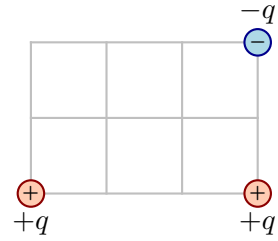
A disk of radius  $R$  has uniform positive surface charge density  $\sigma$ . The disk lies in the  $xy$  plane with its center at the origin. The potential at any point on the  $z$  axis is

$$V = \frac{\sigma}{2\epsilon_0} (\sqrt{R^2 + z^2} - z).$$

- A positive test charge with charge,  $Q$ , and mass,  $m$ , is held at rest at height  $h$  above the center of the disk. Determine an expression for the electrostatic potential energy of the point charge.
- The charge is released. Determine an expression for its velocity when it is infinitely far from the disk.

#### 4 Energy stored in a collection of point charges

Three charged particles are at rest at the illustrated locations. The length of one grid spacing is  $d$ . Determine the work required to assemble this charge arrangement.



#### 5 Energy stored in a charged spherical shell

A hollow spherical shell with radius  $R$  contains a total charge  $Q$ .

- Determine the electric field produced by this at all locations.
- Determine the electrostatic potential at all points assume that  $V = 0$  at infinity.
- Calculate the energy stored in the charge distribution from the electric field that it produces.
- For a two dimensional charge distribution the work done to assemble this is

$$W = \frac{1}{2} \int \sigma V da$$

where the potential is the potential on the surface and the integral is carried out over the surface. Determine this and verify that it gives the same result for the energy stored in the field.

#### 6 Energy stored between plates

Two parallel charged plates each have area  $A$ . One carries charge  $+Q$  and the other carries charge  $-Q$ . They are separated by a distance that is small enough that the plates can be regarded as infinite. The separation between the plates is increased by a factor of  $\alpha$ . Which of the following is true regarding the energy stored in the electric field produced by the plates?

- It stays the same.
- It increases by a factor of  $\alpha$ .
- It increases by a factor of  $\alpha^2$
- It decreases by a factor of  $\alpha$ .
- It decreases by a factor of  $\alpha^2$

Explain your choice.