Phys 311
Fall 2019

Electromagnetic Theory: Homework 10

Due: 24 September 2019

1 Electric field produced by a thick spherical shell

A spherical shell has an inner radius of a and an outer radius of b. Between these ($a \leqslant r' \leqslant b$) charge is distributed according to

$$\rho(r') = \alpha r'^n$$

where α is constant and n is a positive integer.

a) Show that the total charge in the shell is

$$Q = \alpha \frac{4\pi}{n+3}\left(b^{n+3} - a^{n+3}\right).$$

In the following you need to use Gauss' Law to determine electric fields. Your solution *must contain in the following order:*

i) A diagram of the charge distribution.
ii) A simplification of the electric field using symmetry arguments.
iii) An illustration and explanation of what Gaussian surface (the surface that appears in the surface integral) you are using. If this is the same as the surface of the charged shell then all that you will calculate is the field on the surface of the shell. That would be **incomplete and incorrect**. The illustration must indicate the enclosed charge.
iv) Evaluation of the surface integral.
v) Evaluation of the enclosed charge.

Generally the last part is the only part that will vary from one region to another.

b) Determine the electric field for $r \leqslant a$.
c) Determine the electric field for $a \leqslant r \leqslant b$.
d) Determine the electric field for $b \leqslant r$.
e) Check your answer in the limit as $a = 0$ and $n = 0$ against an answer that you can find in the text.

2 Electric field produced by an infinitely long coaxial cylinder

A solid cylinder has radius a and carries a positive charge with uniform charge density, ρ. It is surrounded by a cylindrical shell along the same axis of radius $b > a$. This carries a negative charge with uniform surface charge density σ.

In the following you need to use Gauss' Law to determine electric fields. Your solution *must contain in the following order:*

i) A diagram of the charge distribution.

ii) A simplification of the electric field using symmetry arguments.

iii) An illustration and explanation of what Gaussian surface (the surface that appears in the surface integral) you are using. If this is the same as the surface of the charged cylinder then all that you will calculate is the field on the surface of the cylinder. That would be **incomplete and incorrect**. The illustration must indicate the enclosed charge.

iv) Evaluation of the surface integral.

v) Evaluation of the enclosed charge.

Generally the last part is the only part that will vary from one region to another.

a) Determine an expression for the electric field at all points: within the solid cylinder, between the objects and outside the cylindrical shell.

b) Suppose that the arrangement is electrically neutral, i.e. every section of a given length has zero net charge. Determine a relationship between ρ and σ that ensures this. Use this to simplify the expressions for the field at all locations.

3 Electric field between parallel plates

Two infinite plates are parallel to each other and separated by a distance d. The surface charge density on the upper plate is $+\sigma$ (here $\sigma > 0$) and on the lower plate it is $-\sigma$.

Determine the electric field at any point between and beyond the plates. *(Hint: think about superposition rather than Gauss' or Coulomb's Laws).*

4 Electric field produced by an infinite slab

An infinite plane slab with thickness d consists for two faces at $z = +/2$ and $z = -d/2$. Assume that the slab contains charge with uniform charge density ρ.

In the following you need to use Gauss' Law to determine electric fields. Your solution *must contain in the following order:*

i) A diagram of the charge distribution.

ii) A simplification of the electric field using symmetry arguments.

iii) An illustration and explanation of what Gaussian surface (the surface that appears in the surface integral) you are using. If this is the same as the surface of the charged cylinder then all that you will calculate is the field on the surface of the cylinder. That would be **incomplete and incorrect**. The illustration must indicate the enclosed charge.

iv) Evaluation of the surface integral.

v) Evaluation of the enclosed charge.

Generally the last part is the only part that will vary from one region to another.

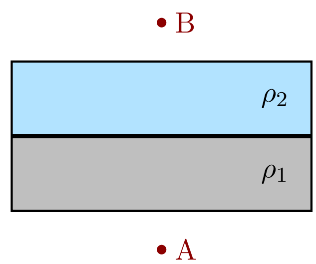
a) Determine the electric field at all points where $z > d/2$.

b) Determine the electric field at all points where $z < -d/2$.

c) Determine the electric field at all points where $-d/2 \leqslant z \leqslant d/2$.

5 Parallel charged slabs

Consider two infinite parallel slabs, each with the same width. One slab has uniform charge density ρ_1 and the other uniform charge density ρ_2. Consider two field locations, A and B, as illustrated.

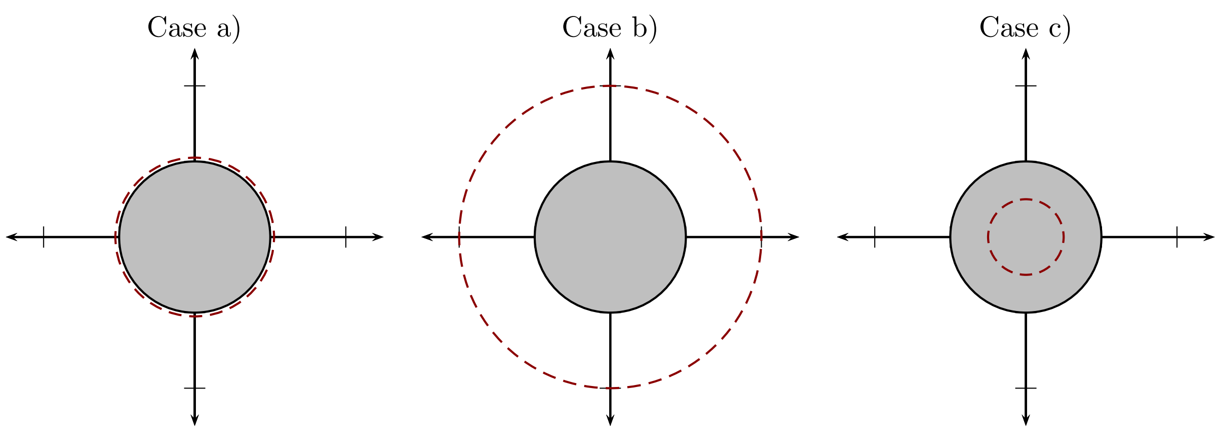


a) How does the magnitude of the field at A compare to that at B? Explain your answer.

b) Suppose that the charges in the two slabs were redistributed between the two slabs so that the charge density throughout the upper slab was the same as that through the lower slab. Would there be any difference in the force exerted on a test charge placed at location A before and after the charge redistribution? Explain your answer.

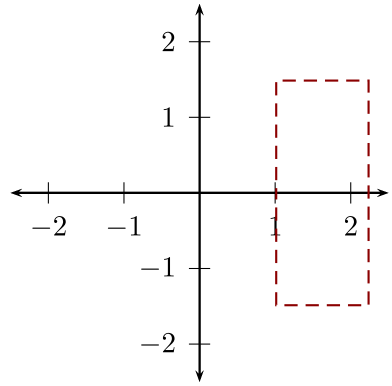
6 Gaussian surfaces

A sphere with radius R carries a charge distribution which is spherically symmetric. This is indicated in the diagram by the shaded circle. In order to compute the electric field at a distance $2R$ from the center of the sphere which Gaussian surface, indicated by the dashed line, would be most useful? Explain your answer.



7 Charge distribution

A charge distribution produces an electric field $\mathbf{E} = Ax^2\hat{\mathbf{x}}$ where $A \geqslant 0$ is a constant. Consider the illustrated rectangular region from $1 \leqslant x \leqslant 2.25$, $-1.5 \leqslant y \leqslant 1.5$ and $0 \leqslant z \leqslant 1$. Is the total electric charge in this region zero or not? Explain your answer.



8 Griffiths, *Introduction to Electrodynamics, 4ed*, 2.18, page 76. *This is a **bonus** problem, worth a total of 5 bonus points. It looks fiendish but using the results of Prob 2.12, careful construction of vectors to any point in the overlap and a little vector algebra quickly gives the result.*