Electromagnetic Theory: Homework 9

Due: 20 September 2019

1 Charge density

A square plate with 2.0 m long sides is centered on the origin with sides parallel to the xy axes. The surface charge density is $\sigma(\mathbf{r}')$ where $\mathbf{r}'$ refers to any location on the plate.

a) If the total charge on the plate is computed, can the result depend on $\mathbf{r}'$, or x', y'?

b) Suppose that $\sigma(\mathbf{r}') = 50\,\text{C/m}^2$ throughout the plate. Determine the total charge Q on the plate. Is $Q = \sigma A$ where A is the area of the plate?

c) Now suppose that

$$\sigma(\mathbf{r}') = \begin{cases} 100\,\text{C/m}^2 & \text{if } 0 \leqslant x' \leqslant 1 \text{ and } 0 \leqslant y' \leqslant 1 \\ 60\,\text{C/m}^2 & \text{if } -1 \leqslant x' \leqslant 0 \text{ and } 0 \leqslant y' \leqslant 1 \\ 40\,\text{C/m}^2 & \text{if } -1 \leqslant x' \leqslant 0 \text{ and } -1 \leqslant y' \leqslant 0 \\ 0\,\text{C/m}^2 & \text{if } 0 \leqslant x' \leqslant 1 \text{ and } -1 \leqslant y' \leqslant 0 \end{cases}$$

Determine the total charge Q on the plate. Is $Q = \sigma A$ where A is the area of the plate?

d) How do the total charges on the plate compare for these two different distributions? Consider the electric fields produced at the center of the plate by each of these two different charge distributions. Are they the same or different?

2 Electric field produced by a charged disk

Consider a disk of radius R and which carries a surface charge distributed according to

$$\sigma(\mathbf{r}') = \alpha s'^n \cos\phi'$$

where s' and ϕ' are cylindrical coordinates for locations on the disk, $n > 0$ is an integer and α is a constant with units of C m^{-2-n}.

a) Determine the total charge on the disk.

b) Determine the electric field at the center of the disk.

3 Electric field produced by a uniformly charged solid sphere

A solid sphere with radius R carries charge with uniform density ρ everywhere inside the sphere. In this exercise you will compute the electric field in two ways. The first is a direct use of Coulomb's law and the second uses Gauss' law.

a) Assuming that the sphere is centered at the origin, consider the electric field at any point along the $+z$ axis. Determine an expression for the contribution to the electric field produced by the segment of the sphere with spherical coordinates r', θ', ϕ'.

b) Set up an integral (*without actually evaluating it*) over all such contributions. By considering the integrals over ϕ', simplify the integral expression for the electric field to show that

$$\mathbf{E}(\mathbf{r}) = \frac{\rho}{2\epsilon_0} \int_0^R \mathrm{d}r' \int_0^\pi \mathrm{d}\theta' \frac{(z - r' \cos\theta')\ r'^2 \sin\theta'}{(z^2 + r'^2 - 2zr' \cos\theta')^{3/2}}\,\hat{\mathbf{z}}.$$

Carry out the integration over θ'. *Hint: You can use MAPLE, Wolfram Alpha or something comparable to evaluate the integral.*

c) Assume that $z > R$, in which case $z \geqslant r'$ for all possible r'. Note that $z^2 + r'^2 - 2zr' = (z - r')^2$. Evaluate the remaining integral over r'.

When $z \geqslant r'$ the square roots become more intricate. We set this case aside and consider using Gauss' Law.

d) Apply Gauss' law to this problem to find the electric field at all points. Express the result in terms of the total charge on the sphere.

4 Electric field produced by a non-uniformly charged solid sphere

Suppose that the charge on a sphere is not uniformly distributed, but the density had the form $\rho = \alpha \cos\theta'$. Explain in detail, without actually calculating, whether you would be able to use Gauss' law to easily determine the field for this distribution.