Electromagnetic Theory: Homework 6

Due: 10 September 2019

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10%, point the errors out and you must submit a corrected assignment by 17 September 2019. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 24 September 2019. This will continue until you **have solved every problem correctly.** If at any stage, you can correct the remaining errors in less than ten minutes, the reduction in grade will only be 5%.

1 Stoke's theorem

Let

$$\mathbf{v} = \frac{y}{2}\,\hat{\mathbf{x}} - \frac{1}{2x^3}\,\hat{\mathbf{y}}$$

and consider the path with straight line segments $(1,1,0) \rightarrow (2,1,0) \rightarrow (2,2,0) \rightarrow (1,2,0) \rightarrow (1,1,0)$. Verify that Stoke's theorem is true for this loop using the flat surface that it encloses.

2 Surface integrals for uniform fields

Consider an arbitrary vector field $\mathbf{v}$. The surface integral $\oint \boldsymbol{\nabla} \times \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ is computed over two surfaces: 1) a disk in the xy plane centered at the origin (normal points along $\hat{\mathbf{z}}$) and 2) a hemisphere whose base is the same as the disk and is above the base (normal is outward). How are the two surface integrals related? *Hint: Don't try to actually evaluate an integral. Think about Stoke's theorem.*

3 Cylindrical unit vectors

Consider two points in the $z = 0$ plane. Denote the point $(1,1,0)$ by P_1 and the point $(1,-1,0)$ by P_2. Is $\hat{\mathbf{s}}$ the same at P_1 as P_2? Is $\hat{\boldsymbol{\phi}}$ the same at P_1 as P_2? Explain your answers.

4 Stoke's theorem: cylindrical coordinates

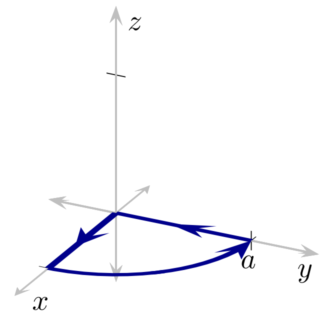
Consider the vector field, in cylindrical coordinates,

$$\mathbf{v} = z\hat{\mathbf{s}} - s\hat{\mathbf{z}}.$$

a) Sketch the vector field in the xz plane, i.e. at locations for which $\phi = 0$. Your sketch should include an s axis, a z axis and the vector field. Determine $\boldsymbol{\nabla} \times \mathbf{v}$ and verify that the result is consistent with your sketch of the vector field.

b) Verify that Stoke's theorem applies to this field by using the path (given in Cartesian coordinates) $(0,0,0) \rightarrow (0,0,1) \rightarrow (1,0,1) \rightarrow (1,0,0) \rightarrow (0,0,0)$.

5 Stoke's theorem: cylindrical coordinates



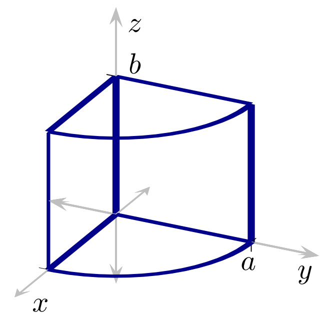
Consider the vector field, in cylindrical coordinates,

$$\mathbf{v} = s\cos\phi\,\hat{\mathbf{s}} - s\sin\phi\,\hat{\boldsymbol{\phi}}$$

a) Determine the line integral along the illustrated path.

b) Verify that Stoke's theorem is true in this case.

6 Divergence theorem: cylindrical coordinates



Consider the vector field, in cylindrical coordinates,

$$\mathbf{v} = s\sin\phi\,\hat{\mathbf{s}} + s\cos\phi\hat{\boldsymbol{\phi}} + z\hat{\mathbf{z}}$$

and the illustrated surface which is a quarter cylinder of radius a and height b.

a) Determine surface integral integral over the entire closed surface.

b) Verify that the divergence theorem is true in this case.