Electromagnetic Theory: Homework 5

Due: 6 September 2019

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10%, point the errors out and you must submit a corrected assignment by 13 September 2019. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 20 September 2019. This will continue until you **have solved every problem correctly.** If at any stage, you can correct the remaining errors in less than ten minutes, the reduction in grade will only be 5%.

1 Fundamental theorem for gradients

Let

$$f(x, y, z) = x^2 y + x y^2 z.$$

a) Determine the gradient of f.

b) Determine the line integral of $\boldsymbol{\nabla} f$ over the line consisting of the straight line segments that run as follows: $(0, 0, 0) \rightarrow (1, 0, 0) \rightarrow (1, 1, 0) \rightarrow (1, 1, 1)$.

c) Check that the fundamental theorem for gradients for this line integral.

2 Divergence theorem

Let

$$\mathbf{v} = -3x^2 y^3 \hat{\mathbf{x}} - 3y^2 x^3 \hat{\mathbf{y}}.$$

Consider the region enclosed by the rectangular box for which $0 \leqslant x \leqslant 2$, $0 \leqslant y \leqslant 2$, and $0 \leqslant z \leqslant 1$.

a) Determine $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ for the entire surface.

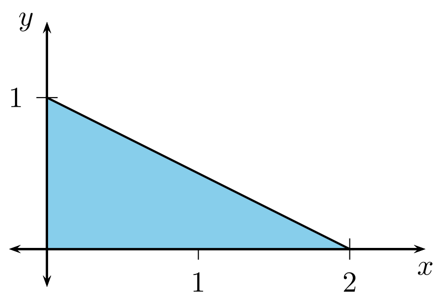
b) Determine $\int \boldsymbol{\nabla} \cdot \mathbf{v} \, \mathrm{d}\tau$ for the region and verify that the divergence theorem is satisfied.

3 Surface integrals and divergence theorem

Let

$$\mathbf{v} = -2xz \hat{\mathbf{x}} + 3xz \hat{\mathbf{z}}.$$

and consider the surface consisting of the "wedge" region cross section in the xy plane is as illustrated and whose top is at $z = 1$ and bottom at $z = 0$.



a) Determine the surface integral for each of the five surfaces.

b) Determine $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ for the entire surface.

c) Determine $\int \boldsymbol{\nabla} \cdot \mathbf{v}\, \mathrm{d}\tau$ for the region and verify that the divergence theorem is satisfied.

4 Uniform vector fields

a) Let $\mathbf{v} = v\hat{\mathbf{x}}$ and consider the sphere centered at the origin. Is $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ positive, negative or zero for the spherical surface? Explain your answer.

b) Consider an arbitrary uniform vector field and *any* closed surface. Is $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ positive, negative or zero for the spherical surface? Explain your answer.