Electromagnetic Theory: Homework 4

Due: 3 September 2019

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10%, point the errors out and you must submit a corrected assignment by 10 September 2019. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 17 September 2019. This will continue until you **have solved every problem correctly.** If at any stage, you can correct the remaining errors in less than ten minutes, the reduction in grade will only be 5%.

1 Charge density

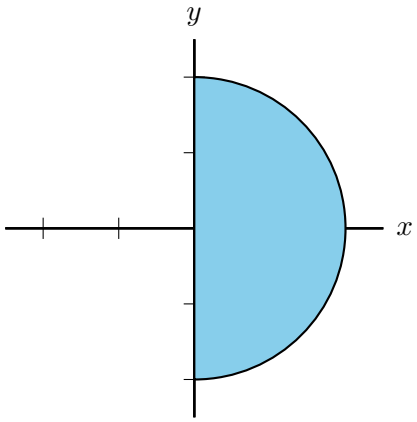
Charge is distributed with density

$$\rho = \frac{q}{a^4}\, x$$

where q is a constant with dimensions of charge.

a) Determine the total charge in the cube with sides of length a and corners at $(0,0,0)$, $(0,0,a)$, $(0,a,0)$, $(0,a,a)$, $(a,0,0)$, $(a,0,a)$, $(a,a,0)$, (a,a,a).

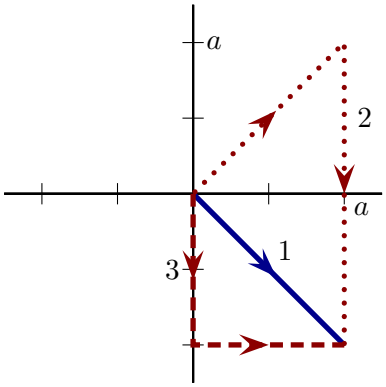
b) Determine $\int \rho \, \mathrm{d}\tau$ for the region which consists of half of a cylinder that is parallel to the z axis, ranges from $z = 0$ to $z = a$, has radius a and whose base in the xy plane is as illustrated.



c) Explain without integrating what the total charge would be in a cylinder of the type described above but whose base is a full circle in the xy plane.

2 Line integrals in two dimensions

Let $\mathbf{v} = ay\hat{\mathbf{x}} - x^2\hat{\mathbf{y}}$. Let a be a positive number. Three paths are indicated in the $z = 0$ plane below.



a) Determine the line integral of $\mathbf{v}$ along line 1.

b) Determine the line integral of $\mathbf{v}$ along line 2: in the xy plane with straight line segments $(0,0) \to (a,a) \to (a,-a)$.

c) Determine the line integral of $\mathbf{v}$ along line 3: in the xy plane with straight line segments $(0,0) \to (0,-a) \to (a,-a)$.

3 Line integrals in three dimensions

Let $\mathbf{v} = y\hat{\mathbf{x}} - x\hat{\mathbf{y}} + xy\hat{\mathbf{z}}$.

a) Determine the line integral of $\mathbf{v}$ along the line with straight segments $(0,0,0) \to (0,0,1) \to (0,1,1) \to (1,1,1)$.

b) Determine the line integral of $\mathbf{v}$ along the line with straight segments $(0,0,0) \to (0,0,1) \to (1,1,1)$.

c) Determine the line integral of $\mathbf{v}$ along the line with straight segments $(0,0,0) \to (1,0,0) \to (1,1,1)$.

d) Determine the line integral of $\mathbf{v}$ along the line with straight segment $(0,0,0) \to (1,1,1)$.

e) Does any line integral from $(0,0,0)$ to $(1,1,1)$ give the same result regardless of the path taken? Explain your answer.

4 Line integrals along a closed loop

Let

$$\mathbf{u} = -y\hat{\mathbf{x}} + x\hat{\mathbf{y}} \quad \text{and}$$
$$\mathbf{v} = x\hat{\mathbf{x}}$$

Consider the line integral of each along the illustrated circular loop. Without actually computing the line integrals, describe whether the line integral of each vector field will be zero or not. Explain your answers. *Hint: sketch the vector fields.*