# Electromagnetic Theory: Homework 3

Due: 30 August 2019

## 1 Divergence and curl of a vector field with three components

Let

$$\mathbf{v} = zx\hat{\mathbf{x}} + xy\hat{\mathbf{y}} + zy\hat{\mathbf{z}}.$$

Determine the divergence and curl of $\mathbf{v}$.

## 2 Radial vector field

Let

$$\mathbf{v} = \frac{\hat{\mathbf{r}}}{r^n}$$

where $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ and $n$ is an integer.

a) Sketch this vector field. Use the sketch describe whether you expect $\boldsymbol{\nabla} \cdot \mathbf{v}$ to be positive, negative or zero. Use the sketch to describe whether you expect $\boldsymbol{\nabla} \times \mathbf{v}$ to be zero or not.

b) Determine $\boldsymbol{\nabla} \cdot \mathbf{v}$. For which values of $n$ is this positive, negative or zero? Do the results result agree your predictions? *Hint: first rewrite* $\mathbf{v}$ *in terms of* $\mathbf{r}$.

c) Determine $\boldsymbol{\nabla} \times \mathbf{v}$. Does the result agree with your prediction?

## 3 Differentiating products

Consider

$$\begin{aligned} \mathbf{A} &= x\hat{\mathbf{x}} + y\hat{\mathbf{y}} \quad \text{and} \\ \mathbf{B} &= y\hat{\mathbf{x}} - x\hat{\mathbf{y}}. \end{aligned}$$

Show, by direct substitution into either side that

$$\boldsymbol{\nabla}(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\boldsymbol{\nabla} \times \mathbf{B}) + \mathbf{B} \times (\boldsymbol{\nabla} \times \mathbf{A}) + (\mathbf{A} \cdot \boldsymbol{\nabla})\,\mathbf{B} + (\mathbf{B} \cdot \boldsymbol{\nabla})\,\mathbf{A}.$$

for these vector fields.

## 4 Gradient and vector fields

Consider the vector fields

$$\begin{aligned} \mathbf{A} &= x\hat{\mathbf{y}} \\ \mathbf{B} &= y\hat{\mathbf{y}} \end{aligned}$$

a) Based on sketches of these vectors fields would you say that either is the gradient of a function? That is, is there some function $f$ so that $\mathbf{A} = \nabla(f)$ and similarly for $\mathbf{A}$. Explain your answer.

b) How could you check precisely if either vector is the gradient of some function?