Electromagnetic Theory: Homework 1

Due: 23 August 2019

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10%, point the errors out and you must submit a corrected assignment by 24 August 2017. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 31 August 2017. This will continue until you **have solved every problem correctly.**

1 Separation vectors

A charge, labeled A, is located at the point $(3, 3, 0)$. Another charge, labeled B, is located at the point $(0, 2, 2)$. A third charge, labeled C, is at the point $(3, 0, 3)$. Let $\mathbf{r}_A$ denote the position vector for A and $\mathbf{r}_B$ denote the position vector for B, etc, ...

a) Express $\mathbf{r}_{\mathrm{A}}, \mathbf{r}_{\mathrm{B}}$ and $\mathbf{r}_{\mathrm{C}}$ in terms of standard basis vectors.

b) Determine expressions for the separation vectors $\vec{\mathscr{r}}_{\text{A to B}}, \vec{\mathscr{r}}_{\text{B to A}}, \vec{\mathscr{r}}_{\text{A to C}}$ and $\vec{\mathscr{r}}_{\text{B to C}}$ in terms of standard basis vectors.

c) Using your results to the previous part, verify that

$$\vec{\mathscr{r}}_{\text{A to C}} = \vec{\mathscr{r}}_{\text{A to B}} + \vec{\mathscr{r}}_{\text{B to C}}$$

and that

$$\vec{\mathscr{r}}_{\text{B to C}} = \vec{\mathscr{r}}_{\text{B to A}} + \vec{\mathscr{r}}_{\text{A to C}}.$$

2 Orthogonal vectors

Two vectors, $\mathbf{A}, \mathbf{B}$ are orthogonal (perpendicular) if and only if $\mathbf{A} \cdot \mathbf{B} = 0$. Consider the vectors:

$$\begin{aligned}
\mathbf{A} &= 4\hat{\mathbf{z}} \\
\mathbf{B} &= 2\hat{\mathbf{x}} + 3\hat{\mathbf{y}} \\
\mathbf{C} &= 3\hat{\mathbf{x}} + 2\hat{\mathbf{y}} \\
\mathbf{D} &= 3\hat{\mathbf{x}} - 2\hat{\mathbf{y}} \\
\mathbf{E} &= 3\hat{\mathbf{x}} - 2\hat{\mathbf{y}} - 4\hat{\mathbf{z}}
\end{aligned}$$

a) Identify all pairs of vectors which are perpendicular to each other.

b) Is there any set of three vectors (from the list above) such that each vector in the set is perpendicular to each other vector in the set?

c) Determine all possible vectors (including any not listed above) that are perpendicular to $\mathbf{B}$.

d) Would it be possible to have a set of four vectors (including any not listed above) such that any two are perpendicular to each other?

3 Vector algebra and geometry

a) A rectangle has sides of length a and b. A *face diagonal* on this rectangle is a line from one corner through the center to the corner opposite. Using vector algebra determine the angle at which the two face diagonals on the rectangle intersect. You should be able to check that your expression is correct for a square.

b) A *body diagonal* in a cube is a line from one corner through the center of the cube to another corner. There are four body diagonals in any cube. Determine the angles between the four body diagonals at the point where they intersect in the center.

4 Vector Triple Product

Let

$$\begin{aligned} \mathbf{A} &= 4\hat{\mathbf{x}} \\ \mathbf{B} &= 2\hat{\mathbf{x}} + 3\hat{\mathbf{y}} \\ \mathbf{C} &= 2\hat{\mathbf{x}} - 3\hat{\mathbf{y}} \end{aligned}$$

Verify that these satisfy the rule

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

by explicitly calculating all the terms on both sides.

5 Vector Triple Associativity

Consider the vector triple product $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$. This would be associative if

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$$

a) Show, by particular choices of three distinct vectors, that there are some vectors such that the triple product is associative.

b) Show, by particular choices of three distinct vectors, that there are some vectors such that the triple product is not associative.