Phys 311
Fall 2019

Electromagnetic Theory: Class Exam I

4 October 2019

Name: ______________________________ **Total:** /40

Instructions

- There are 4 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C}^2/\mathrm{Nm}^2$
Charge of an electron	$e = -1.60 \times 10^{-19}\,\mathrm{C}$

Integrals

$$\int \sin(ax)\sin(bx)\,\mathrm{d}x = \frac{\sin((a-b)x)}{2(a-b)} - \frac{\sin((a+b)x)}{2(a+b)} \quad \text{if } a \neq b$$

$$\int \cos(ax)\cos(bx)\,\mathrm{d}x = \frac{\sin((a-b)x)}{2(a-b)} + \frac{\sin((a+b)x)}{2(a+b)} \quad \text{if } a \neq b$$

$$\int \sin(ax)\cos(ax)\,\mathrm{d}x = \frac{1}{2a}\sin^2(ax)$$

$$\int \sin^2(ax)\,\mathrm{d}x = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int \cos^2(ax)\,\mathrm{d}x = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$$\int x\sin^2(ax)\,\mathrm{d}x = \frac{x^2}{4} - \frac{x\sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2}$$

$$\int x^2\sin^2(ax)\,\mathrm{d}x = \frac{x^3}{6} - \frac{x^2}{4a}\sin(2ax) - \frac{x}{4a^2}\cos(2ax) + \frac{1}{8a^3}\sin(2ax)$$

Question 1

A sphere with radius R contains total charge that is distributed according to the charge density

$$\rho = \alpha r$$

where r is the distance from the center of the sphere and α is a constant.

a) Suppose that the total charge contained within the entire sphere is Q. Determine an expression for Q in terms of α and R.

b) Determine expressions for the electric field at all points *inside* and *outside* the sphere. The expressions for the electric field must be written in terms of Q.

Question 1 continued . . .

/15

Question 2

Someone proposes the following as an electric field (given in cylindrical coordinates) produced by an arrangement of stationary charges:

$$\mathbf{E} = E\hat{\boldsymbol{\phi}}$$

where E is a constant.

a) Sketch the electric field in the xy plane.

b) Describe whether this electric field could arise from a collection of stationary charges or not. Explain your answer.

/10

Question 3

A particular electrostatic charge distribution gives an electric field, described in cylindrical coordinates, of

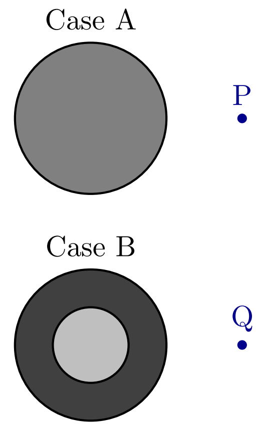
$$\mathbf{E} = \frac{k}{s^2}\hat{\mathbf{s}}$$

where k is a constant. Determine the electrostatic potential at any point, taking the potential at infinity as zero.

/10

Question 4

Two infinitely long cylinders each have the same radius, R and carry charge whose distribution only depends on the radial distance from the cylinder axis. The total charge per unit length of each cylinder is identical. However, in cylinder A it is uniformly distributed and in cylinder B, the charge density increases with distance from the center of the cylinder. Consider the electric fields at points each a distance $2R$ from the cylinder axis in each case. Is the field at point Q the same as, larger than or smaller than the field at point P? Explain your answer.



/5