## Phys 111: Supplementary Exercises

## One dimensional kinematics

## 1 Motion diagrams: horizontal motion

A car moves to the right. For an initial period it slows down and after that it speeds up. Which of the following (choose one) best represents its location as time passes?


Briefly explain your choice.

2 Motion diagrams, speed and velocity
Motion diagrams for two carts are as illustrated.

a) Is the speed of the red cart the same as or different to that of the blue cart? Explain your answer.
b) Is the velocity of the red cart the same as or different to that of the blue cart? Explain your answer.

## 3 Motion diagrams and velocity

A car moves from left to right and its position, measured in meters, is recorded every 5.0 s . The resulting
 motion diagram is illustrated.
a) Determine the average velocity from 0 s to 5.0 s .
b) Determine the average velocity from 5.0 s to 10.0 s .
c) Determine the average velocity from 5.0 s to 15.0 s .
d) Determine the average velocity from 0 s to 15.0 s . Is this the average of the two velocities in the interval $0 \mathrm{~s} \rightarrow 5.0 \mathrm{~s}$ and $5.0 \mathrm{~s} \rightarrow 15.0 \mathrm{~s}$ ?

## 4 Average speed and average velocity

Determine the average speed and the average velocity for cars moving in a straight line as follows:
a) Red car moves right for 50 m in 10.0 s .
b) Blue car moves right for 16 m in 8.0 s and then continues right for 34 m in 2.0 s .
c) Red car moves right for 30 m in 5.0 s reverses and moves left for 20 m in 5.0 s .

## 5 Average velocity

The following objects lie along a straight line: a bicycle, a coffee cup and a soccer ball. The distance from the coffee cup to the bicycle is 400 m and from the cup to the ball is 500 m . A man starts at the cup
 and travels in a straight line to the ball. This takes 200 s . A dog is initially at the cup and runs at constant speed to the bicycle, taking 50 s to do so. The dog immediately turns around and runs to ball; this takes the dog an additional 150 s . Consider the entire trip from the cup to the ball for each. Who has the larger average velocity for this entire trip? Explain your answer.

## 6 Average velocity and displacement down a field

Three people, Alice, Bob and Charlie leave one end of a 100 m long field at the same instant and travel in the same direction toward the other end. Alice travels at a constant speed of $8.0 \mathrm{~m} / \mathrm{s}$ for half the distance and then at $4.0 \mathrm{~m} / \mathrm{s}$ for the remaining half. Bob travels at a constant speed of $10.0 \mathrm{~m} / \mathrm{s}$ for half the distance and then at $2.0 \mathrm{~m} / \mathrm{s}$ for the remaining half. Charlie travels for half of the time at a constant speed of $8.0 \mathrm{~m} / \mathrm{s}$ for the remaining half of the time at a constant speed of $4.0 \mathrm{~m} / \mathrm{s}$. Determine the average velocity of each and the order in which they arrive at the other end of the field.

## 7 Average velocity, speed for motion with changing directions

a) A person takes a trip, first walking 30 m to the right in 5.0 s and then returning to his starting point in another 10 s . Determine the average velocity of the person for the entire trip.
b) Anna takes a trip in two stages. First she moves 100 m right in 40 s . She briefly stops and then she moves 120 m to the left in another 60 s . Determine her average velocity for each stage of the trip and also for the entire trip.
c) Bill takes a trip in two stages. First he moves 200 m left in 100 s . He briefly stops and then he moves 150 m to the right in another 100 s . Determine his average velocity for each stage of the trip and also for the entire trip.

## 8 Average velocity and direction of motion

a) An object moves in such a way that during a certain period the average velocity of the object is negative. Is it possible that at the end of the period the object is to the right of the origin? Explain your answer.
b) An object moves in such a way that during a certain period the average velocity of the object is positive. Is it possible that at some point during the period the object moves to the left? Explain your answer.

## 9 Motion diagrams and position vs. time graphs

A car moves from left to right and its position, measured in meters, is recorded every 5.0 s . The resulting
 motion diagram is illustrated.
a) Produce a table of numerical data for position versus time for the car for the duration of the motion.
b) Produce a position versus time graph for the car for the duration of the motion. This graph must be drawn by hand using axes that are clearly labeled.

## 10 Motion diagrams and position vs. time graphs

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 motion diagram is illustrated.
a) Produce a table of numerical data for position versus time for the car for the duration of the motion.
b) Produce a position versus time graph for the car for the duration of the motion. This graph must be drawn by hand using axes that are clearly labeled.

## 11 Ant and bug on a stick

An ant and a bug walk along a straight stick. The graph illustrates their position vs. time. Answer the following, explaining each answer.
a) When and where, if ever, are the ant and the bug at the same location?
b) When, if ever, do the ant and the bug have the same speed?
c) When, if ever, does the ant move faster than the bug?

## 12 Ant on a stick

An ant walks along a straight stick. The graph illustrates the ant's position vs. time. Answer the following, giving explanations for each answer.
a) During which times is the ant moving right? During which times is it moving left?
b) When, if ever, is the velocity of the ant $0 \mathrm{~m} / \mathrm{s}$ ?
c) How does the speed of the ant at 1.0 s compare to its speed at 4.0 s ?
d) How does the velocity of the ant at 1.0 s compare to its velocity at 4.0 s?

## 13 Tick on a stick

A tick walks along a straight stick. The graph illustrates the tick's position vs. time.
a) Determine the velocity of the tick at 0.5 s .
b) Determine the velocity of the tick at 1.5 s .
c) Determine the velocity of the tick at 3.0 s .
d) Determine the velocity of the tick at 4.5 s .

## 14 Bug on a stick

A bug walks along a straight stick. The graph illustrates the bug's position vs. time. In each of the following, explain your answers.
a) Does the bug ever return to the position that it occupied initially $(t=0 \mathrm{~s})$ ? If so, when?
b) Does the bug ever reverse direction? If so when?
c) Does the bug ever have zero velocity? If so, when?

## 15 Ant and bug on a stick

An ant and a bug walk along straight sticks. The solid graph illustrates the ant's position vs. time. The dashed graph indicates the bug's position vs. time. Answer the following, giving explanations for each answer.
a) At what time(s) are the ant and bug at the same location?
b) Which is moving faster at 2 s ?
c) Do the ant and bug ever have the same velocity? If so when?




## 16 Wandering ant

An ant walks along a straight stick. The graph illustrates the ant's velocity vs. time. Answer the following, giving explanations for each answer.
a) How does the magnitude of the ant's acceleration at 1.5 s compare (larger, smaller, same) to its acceleration at 2.5 s ?
b) How does the magnitude of the ant's acceleration at 0.5 s compare (larger, smaller, same) to its acceleration at 2.5 s ?
c) How does the magnitude of the ant's acceleration at 0.5 s compare (larger, smaller, same) to its acceleration at 4.0 s ?

## 17 Acceleration sign

A bicycle can move east (positive) or west (negative).
a) If the bicycle moves east can the acceleration be negative? Explain your answer.
b) If the bicycle moves west can the acceleration be positive? Explain your answer.

## 18 Ant and bug on a stick

An ant and a bug walk along straight sticks. The solid graph illustrates the ant's position vs. time. The dashed graph indicates the bug's position vs. time. For the bug, and separately for the ant, which of the following is true during the period from $0 s$ to $4 s$ ? Explain your answers.
i) Acceleration is zero at all times.
ii) Acceleration is positive at all times.
iii) Acceleration is negative at all times.
iv) Acceleration is first positive and later negative.
v) Acceleration is first negative and later positive.

## 19 Person walking left and right

A person walks along a horizontal line. Positions are marked in meters along the line with positive numbers to the right of the origin and negative to the left. His velocity is recorded at equally spaced intervals in time. The data is:

| Time in s | Velocity in $\mathrm{m} / \mathrm{s}$ |
| :---: | :---: |
| 10.00 | -1.00 |
| 10.10 | -0.80 |
| 10.20 | -0.60 |
| 10.30 | -0.40 |
| 10.40 | -0.20 |
| 10.50 | 0.00 |
| 10.60 | 0.20 |
| 10.70 | 0.40 |
| 10.80 | 0.60 |
| 10.90 | 0.80 |

a) During which period is the person moving left? During which period is the person moving right?
b) Determine the person's acceleration while moving left.
c) Determine the person's acceleration while moving right.
d) Does the person's acceleration appear to change during the period from 10.10 s to 10.90 s ?
e) At what moment does the person reverse direction? According to the data is the acceleration zero or not at this moment?

## 20 Bungee jumper

A bungee jumper falls downward stretching the cord, reaching a low point, after which the cord pulls him up again. His velocity is recorded at equally spaced intervals in time. The data is:

| Time in s | Velocity in $\mathrm{m} / \mathrm{s}$ |
| :---: | :---: |
| 10.0 | -20.0 |
| 10.5 | -15.0 |
| 11.0 | -10.0 |
| 11.5 | -5.0 |
| 12.0 | 0.0 |
| 12.5 | 5.0 |
| 13.0 | 10.0 |
| 13.5 | 15.0 |
| 14.0 | 20.0 |

a) During which period is the man falling? When is he rising?
b) By how much does the man's velocity change per second? Is this change constant throughout the recorded motion?
c) Determine the man's acceleration while he is falling and also while he is rising. Are these accelerations the same or not?
d) What is the man's acceleration (according to the data) at his low point?

## 21 Moving man

Go to the moving man animation at:

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http://phet.colorado.edu/en/simulation/moving-man
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Run the moving man animation. Click on the charts tab. Set the position to 0.00 m , the velocity to $-5.00 \mathrm{~m} / \mathrm{s}$ and the acceleration to $2.00 \mathrm{~m} / \mathrm{s}^{2}$. Run the animation, stopping it just before the man hits the wall. The animation will have recorded the motion. Check the playback button at the bottom. You can slide the light blue bar left and right to get data for the motion. Gray zoom icons at the right will let you rescale the charts.
a) Consider the interval from 2.0 s to 3.0 s . Describe the motion verbally during this time.
b) How does the speed of the man at 2.0 s compare to that at 3.0 s ? Explain your answer.
c) How does the velocity of the man at 2.0 s compare to that at 3.0 s ? Explain your answer.
d) Will the average acceleration over the interval from 2.0 s to 3.0 s be positive, negative or zero? Explain your answer.
e) If the acceleration is not zero, does it vary during this interval? Explain your answer.
f) Determine the average acceleration over the interval from 2.0 s to 3.0 s .

## 22 Ant and tick on a stick

An ant, a tick and a bug walk along straight sticks. The solid graph illustrates the ant's velocity vs. time. The dashed graph indicates the tick's velocity vs. time. The dotted line indicates the bug's velocity versus time.

a) Which of the following is true during the period from $0 s$ to $4 s$ ? Explain your answer.
i) The acceleration of the ant is the same as that of the tick.
ii) The acceleration of the tick is always larger than that of the ant.
iii) The acceleration of the tick is always smaller than that of the ant.
iv) The acceleration of the tick is sometimes smaller than that of the ant, sometimes larger.
b) Which of the following is true during the period from $0 s$ to $4 s$ ? Explain your answer.
i) The acceleration of the ant is the same as that of the bug.
ii) The acceleration of the bug is always larger than that of the ant.
iii) The acceleration of the bug is always smaller than that of the ant.
iv) The acceleration of the bug is sometimes smaller than that of the ant, sometimes larger.

## 23 Cyclist versus skateboarder

A skateboarder moves down a gentle slope. She passes a cyclist, who is at rest and at this moment the skater's speed is $4.0 \mathrm{~m} / \mathrm{s}$. At an instant 10 s later the skater's speed is $10.0 \mathrm{~m} / \mathrm{s}$ and the cyclist's is $8.0 \mathrm{~m} / \mathrm{s}$. During this period, who has the greater acceleration? Explain your answer.

## 24 Person moving with constant acceleration

A person is initially at rest and subsequently moves right with a constant acceleration. The person's reaches speed $6.0 \mathrm{~m} / \mathrm{s}$ at a point 9.0 m to the right of the starting location. The aim of this exercise will be to determine the time taken to reach this point. A first step will be to determine the acceleration of the person.
a) Sketch the situation, illustrating the person at the two instants described above.

List all relevant variables for the two instants:

$$
\begin{array}{ll}
t_{\mathrm{i}}= & t_{\mathrm{f}}= \\
x_{\mathrm{i}}= & x_{\mathrm{f}}= \\
v_{\mathrm{i}}= & v_{\mathrm{f}}=
\end{array}
$$

b) Determine the acceleration by selecting one of the kinematic equations, substituting and solving for $a$.
c) Using a different kinematic equation, find the time that it takes the person to reach speed $6.0 \mathrm{~m} / \mathrm{s}$.
d) Suppose that you had tried to find the time taken to reach speed $6.0 \mathrm{~m} / \mathrm{s}$ by using

$$
v=\frac{\Delta x}{\Delta t} \Rightarrow 6.0 \mathrm{~m} / \mathrm{s}=\frac{9.0 \mathrm{~m}}{\Delta t} .
$$

What time does this give? Does it agree with the answer that you obtained to the previous part? Is it correct?

## 25 Avoid the wall!

A skateboarder slides toward a wall. Initially the skateboarder is 18 m left of the wall and moving with speed $6.0 \mathrm{~m} / \mathrm{s}$ to the right. The aim of this exercise will be to determine the minimum acceleration to barely avoid hitting the wall.
a) Sketch the situation, illustrating the skateboarder at the initial instants and the instant just before reaching the wall.
List all relevant variables for the two instants:

$$
\begin{array}{ll}
t_{\mathrm{i}}= & t_{\mathrm{f}}= \\
x_{\mathrm{i}}= & x_{\mathrm{f}}= \\
v_{\mathrm{i}}= & v_{\mathrm{f}}=
\end{array}
$$

b) Determine the acceleration by selecting one of the kinematic equations, substituting and solving for $a$.
c) Use one of the kinematic equations to determine the time that it takes for the skateboarder to reach the wall.
d) Would the equation

$$
v=\frac{\Delta x}{\Delta t} \Rightarrow 6.0 \mathrm{~m} / \mathrm{s}=\frac{18 \mathrm{~m}}{\Delta t}
$$

allow one to find the time taken to reach the wall correctly? Why or why not?
e) Set up the moving man animation at:
http://phet.colorado.edu/en/simulation/moving-man
and run this to check your prediction. In order to verify that you have done this, use the animation to provide the times at which the man is 10 m to the left of the wall.

## 26 Braking car

A car travels to the right with speed $30.0 \mathrm{~m} / \mathrm{s}$ (about 67 mph ). The car brakes and slows to a stop with constant acceleration. It does this in a distance of 80.0 m (about 260 ft ). The aim of this exercise is to find the time taken to do this.
a) Sketch the situation, illustrating the car at two key instants. List all relevant variables for the two instants.
b) Determine the acceleration using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for $a$. (is acceleration positive or negative?)
Answer: - $5.63 \mathrm{~ms}^{2}$
c) Using a different kinematic equation, find the time that it takes the car to stop. Write down the equation, substitute from your list of variables and solve for time.
Answer: 5.33 s

## 27 Accelerating aircraft

An aircraft accelerates from rest to a speed of $120.0 \mathrm{~m} / \mathrm{s}$. It does this with a constant acceleration of $4.00 \mathrm{~m} / \mathrm{s}^{2}$. The aim of this exercise is to find the distance traveled by the aircraft during this process.
a) Sketch the situation, illustrating the aircraft at two key instants.

List all relevant variables for the two instants and list the acceleration:
b) Determine the distance traveled by the aircraft using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the relevant variable.
Answer: 1800 m
c) Using a different kinematic equation, find the time that it takes for the aircraft to reach this speed. Write down the equation, substitute from your list of variables and solve for time.
Answer: 30 s
d) Would it have worked to use $v=\frac{\Delta x}{\Delta t} \Rightarrow 120.0 \mathrm{~m} / \mathrm{s}=\frac{1800 \mathrm{~m}}{\Delta t}$ ?

## 28 Accelerating cart

A cart travels in one direction. At an initial instant it passes the 4.0 m mark while traveling to the right with speed $15 \mathrm{~m} / \mathrm{s}$.
a) Determine the location of the cart at an instant 6.0 s later if its acceleration is $3.0 \mathrm{~m} / \mathrm{s}^{2}$.
b) Determine the location of the cart at an instant 6.0 s later if its acceleration is $-3.0 \mathrm{~m} / \mathrm{s}^{2}$.

## 29 Reversing cart

At an initial instant a cart travels to the left with speed $12 \mathrm{~m} / \mathrm{s}$. Subsequently the cart's acceleration is $4.0 \mathrm{~m} / \mathrm{s}^{2}$.
a) How long does it take for the cart to reverse its direction of travel?
b) How far does the cart travel before it reverses its direction of travel?

## 30 Runway design

The takeoff speed for an aircraft under certain conditions is $260 \mathrm{~km} / \mathrm{h}$. During takeoff, one wants the acceleration to be as low as possible.
a) Suppose that the runway has length 1500 m . Determine the aircraft's minimum acceleration so that it can take off on this runway.
b) Suppose that the acceleration is only $0.20 g=1.96 \mathrm{~m} / \mathrm{s}^{2}$. Determine the minimum length of the runway so that the aircraft could take off successfully.

## 31 Jumping cat

A cat jumps, launching itself vertically. During it's subsequent motion up and down, air resistance is negligible. In the following let the upward direction be positive.
a) Which of the following is true in the period after the cat's feet have left the ground and before the cat reaches its highest point above the ground?
i) Acceleration is zero at all times.
ii) Acceleration is positive at all times.
iii) Acceleration is negative at all times.
b) Which of the following is true in the period while the cat drops back to the ground?
i) Acceleration is zero at all times.
ii) Acceleration is positive at all times.
iii) Acceleration is negative at all times.
c) Which of the following is true when the cat is at is highest point above the ground?
i) Acceleration is zero at all times.
ii) Acceleration is positive at all times.
iii) Acceleration is negative at all times.

## 32 Ball launched vertically

A ball is launched vertically from Earth's surface with speed $10 \mathrm{~m} / \mathrm{s}$. The aim of the first part of this exercise is to determine the maximum height reached by the ball and time taken to reach the maximum height.
a) Sketch the situation, illustrating the ball at two key instants. List all relevant variables, including the acceleration, for the two instants.
b) Determine the maximum height reached by the ball by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the maximum height variable.
Answer: 5.1 m
c) Determine the time taken to reach the maximum height by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the maximum height variable.
Answer: 1.0 s
The second part of this exercise aims to find the speed of the ball just before returning to the ground using the fall from its maximum height.
d) Sketch the situation for the falling ball, illustrating the ball at two key instants. List all relevant variables, including the acceleration, for the two instants.
e) Determine the speed of the ball just before hitting the ground the ball by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the velocity variable.
Answer: velocity: $-10 \mathrm{~m} / \mathrm{s}$, speed: $10 \mathrm{~m} / \mathrm{s}$.

## 33 Parachuting package

A package is released from rest at a height of 100 m above Earth's surface. It falls freely until it is 40 m above Earth's surface. At that instant it deploys a parachute and after this is falls with a constant speed. The aim of this exercise is to determine the time taken to reach Earth.
To do this we will calculate the time taken for the free fall motion, $\Delta t_{A}$, and separately that time taken to fall with the parachute open, $\Delta t_{B}$.
The aim of the first part of this exercise is to determine $\Delta t_{A}$.
a) Sketch the situation, illustrating the ball at two key instants that will allow one to determine the time for the free fall portion of the motion.
List all relevant variables for the two instants and the acceleration.
b) Determine $\Delta t_{A}$ by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for $\Delta t_{A}$.
Answer: 3.5 s .
The second part of this exercise aims to find $\Delta t_{B}$.
c) Sketch the situation for the falling ball, illustrating the ball at two key instants that will allow one to determine the time for the parachuting portion of the motion.
List all relevant variables for the two instants and the acceleration.
d) There is one quantity that one can obtain from the free fall part of the motion that will be needed to analyze the parachuting portion of the motion. Identify and compute this and insert it in to the list of variables from the previous part.
e) Now determine $\Delta t_{B}$ by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for $\Delta t_{B}$.
Answer: 1.2 s .
f) Determine the total time taken to fall.

Answer: 4.7 s .

## 34 Thrown rock

A rock is thrown vertically down from a bridge. It leaves the hand with speed $20 \mathrm{~m} / \mathrm{s}$ and hits the water 15 m below. Determine its speed at the instant before it hits the water.
Answer: $26 \mathrm{~m} / \mathrm{s}$

## 35 Diver

A diver stands on a diving board that is 10 m above the surface of the water. The diver launches herself vertically up, leaving the board with a speed of $5.0 \mathrm{~m} / \mathrm{s}$.
a) Determine the time taken for the diver to hit the water. Answer: 2.0 s
b) Determine the diver's speed when she hits the water. Answer: 15 s

## 36 Penny and a well

A penny is held at rest at the top of a well. The penny is released, falls freely and takes 1.75 s to hit the water below. Determine the depth (distance from the top to the water) of the well.
Answer: 15 m

## Vectors

## 37 Vector addition and subtraction

Two displacement vectors, $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$, are illustrated.
a) Sketch $\overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}$. Determine the magnitude and direction of $\overrightarrow{\mathbf{C}}$.
b) Let $A$ be the magnitude of $\overrightarrow{\mathbf{A}}$, let $B$ be the magnitude of $\overrightarrow{\mathbf{B}}$ and $C$ be the magnitude of $\overrightarrow{\mathbf{C}}$. In this case do these magnitudes satisfy $C=A+B$ ? Explain your asnwer


## 38 Adding two vectors

Consider displacement vectors $\overrightarrow{\mathbf{A}}$, whose magnitude is 4.0 m , and $\overrightarrow{\mathbf{B}}$, whose magnitude is 3.0 m . Their directions are not specified. Let $\overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}$.
a) What is the maximum possible magnitude for $\overrightarrow{\mathbf{C}}$ ? How must the vectors be arranged to give this?
b) What is the minimum possible magnitude for $\overrightarrow{\mathbf{C}}$ ? How must the vectors be arranged to give this?

## 39 Vector addition and subtraction

Two displacement vectors, $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$, are illustrated.
a) Sketch $\overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}$. Determine the magnitude and direction of $\overrightarrow{\mathbf{C}}$.
b) Sketch $\overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}$. Determine the magnitude and direction of $\overrightarrow{\mathbf{C}}$.
c) Sketch $\overrightarrow{\mathbf{C}}=2 \overrightarrow{\mathbf{A}}-\frac{1}{2} \overrightarrow{\mathbf{B}}$. Determine the magnitude and direction of $\overrightarrow{\mathbf{C}}$.


## 40 Vector addition: graphical and algebraic method

Displacement vectors, $\overrightarrow{\mathbf{A}}, \overrightarrow{\mathbf{B}}$, and $\overrightarrow{\mathbf{C}}$ are illustrated. Let $\overrightarrow{\mathbf{D}}=\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}}$.
a) Using the graph sheet below, determine $\overrightarrow{\mathbf{D}}$ graphically via the head-to-tail method. Use the result to determine the magnitude of $\overrightarrow{\mathbf{D}}$.
b) List the horizontal and vertical components of each of $\overrightarrow{\mathbf{A}}, \overrightarrow{\mathbf{B}}$, and $\overrightarrow{\mathbf{C}}$ and use these components to determine the components of $\overrightarrow{\mathbf{D}}$. Use the result to determine the magnitude of $\overrightarrow{\mathbf{D}}$.



## 41 Vector algebra using components

Displacement vectors, $\overrightarrow{\mathbf{A}}, \overrightarrow{\mathbf{B}}, \overrightarrow{\mathbf{C}}$, and $\overrightarrow{\mathbf{D}}$ are illustrated. Their magnitudes are $A=2.0 \mathrm{~m}, B=$ $2.5 \mathrm{~m}, C=3.0 \mathrm{~m}$ and $D=1.5 \mathrm{~m}$.
a) Determine the $x$ and $y$ components for each vector.
b) Determine the $x$ and $y$ components for $\overrightarrow{\mathbf{E}}=$ $\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}$. Determine the magnitude of $\overrightarrow{\mathbf{E}}$.
c) Determine the $x$ and $y$ components for $\overrightarrow{\mathbf{E}}=$ $\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}}$. Determine the magnitude of $\overrightarrow{\mathbf{E}}$.
d) Determine the $x$ and $y$ components for $\overrightarrow{\mathbf{E}}=$ $\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{D}}$. Determine the magnitude of $\overrightarrow{\mathbf{E}}$.

e) Determine the $x$ and $y$ components for $\overrightarrow{\mathbf{E}}=\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}}+\overrightarrow{\mathbf{D}}$. Determine the magnitude of $\overrightarrow{\mathbf{E}}$.

## 42 Vector subtraction using components

Displacement vectors, $\overrightarrow{\mathbf{A}}, \overrightarrow{\mathbf{B}}$ are illustrated. Their magnitudes are $A=3.16 \mathrm{~m}$ and $B=$ 2.83 m .
a) Determine the components of $\overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}$. Determine the magnitude of $\overrightarrow{\mathbf{C}}$.
b) Determine the components of $\overrightarrow{\mathbf{C}}=4 \overrightarrow{\mathbf{A}}-$ $3 \overrightarrow{\mathbf{B}}$. Determine the magnitude of $\overrightarrow{\mathbf{C}}$.


## Two dimensional kinematics

## 43 Velocity and acceleration vectors

Two particles move along the illustrated trajectories. Their locations and speeds are illustrated at instants 0.25 s apart. For each case determine the acceleration vector for the particle.
a)

b)


## 44 Acceleration vector

a) A hockey puck slides along a horizontal surface toward a board, hitting it at an angle and bouncing off with unchanged speed. The view from above is as illustrated. Draw the velocity vectors of the puck just before and just after hitting the board, use these to draw the vector $\Delta \overrightarrow{\mathbf{v}}$, and use the result to draw the direction of the acceleration vector.

b) If the puck traveled backwards along the same path (i.e. reversed direction), what would the direction of the acceleration vector be?

## 45 Ants moving along a curved path

Various ants follow the same path on a horizontal surface, starting at point 1. The path is as illustrated. Ant A moves with a constant speed, ant B gradually speeds up and ant C gradually slows.
a) Draw the velocity vector at points $1,2,3$ and 4 .

b) Does any of the ants have zero acceleration at all times? Explain your answer.

## 46 Dropping a coin on a ship

A person is on the deck of a ship. While the ship is at rest in a harbor, the person drops a coin from rest. The person marks the spot where the coin lands. Later the ship sails with constant speed in a straight line and the person returns to the same spot and drops the coin again. Does the coin land at the original mark, in front of it or behind it? Explain your answer.

## 47 Jumping monkey

A monkey jumps leaving the Earth at an angle of $45^{\circ}$ from the surface. Which of the following is true when the monkey reaches the highest point along the trajectory? Explain your answer.
i) The acceleration of the monkey is zero.
ii) The acceleration of the monkey is horizontal in the forwards direction.
iii) The acceleration of the monkey is horizontal in the backwards direction.
iv) The acceleration of the monkey is vertical downward.
v) The acceleration of the monkey is vertical upward.

## 48 Red vs. blue ball

Two balls, one red and the other blue, are on a horizontal table. The are made to roll off the table. At the instants that they leave the speed of the blue ball is four times that of the red ball. Which of the following is true regarding the times taken to hit the floor on which the table stands? Explain your answer.
i) The time taken for blue ball is a quarter of that for the red ball.
ii) The time taken for blue ball is half that for the red ball.
iii) The time taken for blue ball is the same as that for the red ball.
iv) The time taken for blue ball is twice that for the red ball.
v) The time taken for blue ball is four times that for the red ball.

## 49 Running off a roof

A person runs with speed $8.0 \mathrm{~m} / \mathrm{s}$ off a flat roof that is 3.0 m above the ground. First suppose that the person travels horizontally at the moment that he leaves the roof. Determine how far horizontally from the edge of the roof the person will land.
a) Sketch the situation with the "earlier" instant being that at which the person leaves the roof and the "later" instant being the moment just before the person hits the ground.

List as many of the variables as possible. Use the format:

$$
\begin{array}{rlrl}
t_{i} & = & t_{f}= \\
x_{i} & = & & x_{f}= \\
y_{i} & = & y_{f}= \\
v_{i x} & = & v_{f x}= \\
v_{i y} & = & & v_{f y}= \\
a_{x} & = & & a_{y}=
\end{array}
$$

b) Sketch the velocity vector at the earlier moment and use this to determine the components of $\overrightarrow{\mathbf{v}}_{0}$. Enter these in the list above.
c) Identify the variable needed to answer the question of the problem. Select and write down a kinematic equation that contains this variable and attempt to solve it.

You should see to solve the variable describing the horizontal position, you first need the value for another, currently unknown variable. Which variable is this?
d) Use the vertical aspects of the object's motion to solve for this other unknown variable and use this result to answer the question of this problem.

Suppose that the person ran and jumped from the building at an angle of $30^{\circ}$ above the horizontal. This will change how far the person travels. Before answering that question, we ask, what is the maximum height above the ground reached by the person for this running jump?
e) Sketch the velocity vector at the earlier moment and use this to determine the components of $\overrightarrow{\mathbf{v}}_{0}$. Reconstruct the list of variables for the problem.
f) Sketch the velocity vector at the instant when the person reaches his highest point. Use this to add additional information to the list of variables for the problem.
g) Use the kinematic equations to determine the maximum height that the person reaches.
h) Determine the speed of the person at the maximum height.
i) Determine how far horizontally from the edge of the roof the person will land.

## 50 Projectile motion range

A person runs with speed $8 \mathrm{~m} / \mathrm{s}$ off a flat roof that is 3.0 m above the ground. The person can launch himself at various angles and the purpose of this exercise is to determine the speed with which the person hits the ground and where on the ground the person lands (for various angles of launch). First suppose that the person travels horizontally at the moment that he leaves the roof. It was found that the person lands 6.3 m from the base of the building.
a) Sketch the situation with the "earlier" instant being that at which the person leaves the roof and the "later" instant being the moment just before the person hits the ground. List as many of the variables as possible. Use the format

$$
\begin{aligned}
& t= \\
x_{0}= & x= \\
y_{0}= & y= \\
v_{0 x}= & v_{x}= \\
v_{0 y}= & v_{y}= \\
a_{x}= & a_{y}=
\end{aligned}
$$

b) Draw the velocity vector at the earlier instant and use this to determine the components of $\overrightarrow{\mathbf{v}}_{0}$. Enter these in the list above.
c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero.
d) The speed of the object is the magnitude of the velocity vector. So to determine the speed, you will first need to determine the components of the velocity vector. Use the kinematic equations to determine the $x$ and $y$ components of the velocity at the later instant. You must start by writing the equation that you use, before substituting.
e) Sketch the velocity vector at the instant just before the person hits the ground and indicate the values of the $x$ and $y$ components on the sketch. Use this to determine the speed of the person just before hitting the ground.

Now suppose that the person travels with the same speed but launches himself at an angle of $45^{\circ}$ from the roof.
f) Using the same "earlier" and later instants as before, list as many of the variables as possible.
g) Draw the velocity vector at the earlier moment and use this to determine the components of $\overrightarrow{\mathbf{v}}_{0}$.
h) Use the kinematic equations to determine the $x$ and $y$ components of the velocity at the later instant.
i) Sketch the velocity vector at the instant just before the person hits the ground and indicate the values of the $x$ and $y$ components on the sketch. Use this to determine the speed of the person just before hitting the ground.
j) Determine the time taken to hit the ground.
k) Determine how far the person lands from the building.

## 51 Jumping over a ditch

A dog attempts to jump over a ditch, which is 2.0 m wide. The dog launches itself from one edge of the ditch with speed $5.0 \mathrm{~m} / \mathrm{s}$ at an angle of $30^{\circ}$ from the horizontal. The aim of this exercise is to determine whether the dog will reach the other side of the ditch. First we will find out whether the dog reaches its maximum height before or after it is above the middle of the ditch.
a) Sketch the situation with the "earlier" instant being that at which the dog launches and the "later" instant being the moment when it reaches its highest point. List as many of the variables as possible. Use the format

$$
\begin{array}{rlrl}
t_{i} & = & t_{f}= \\
x_{i} & = & & x_{f}= \\
y_{i} & = & y_{f}= \\
v_{i x} & = & v_{f x}= \\
v_{i y} & = & & v_{f y}= \\
a_{x} & = & & a_{y}=
\end{array}
$$

b) Draw the velocity vector at the earlier instant and use this to determine the components of $\overrightarrow{\mathbf{v}}_{0}$. Enter these in the list above.
c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero and enter as much information about these in the list above.
d) Determine the horizontal distance traveled by the dog by the time that it reaches its maximum height.
e) As the dog descends from its maximum height back to the ground, how much further does it travel? Does it reach the other side of the ditch?

## 52 Launching off a ski ramp

A ski ramp is arranged as illustrated. A skier launches off the ramp with a speed of $15 \mathrm{~m} / \mathrm{s}$. Initially the aim of this exercise is to determine the maximum height reached by the skier and the velocity at this point. A later goal is to determine the distance at which the skier lands from the bottom of the ramp.

a) Sketch the situation with the "earlier" instant being that at which the skier launches and the "later" instant being the moment when she reaches its highest point. List as many of the variables as possible. Use the format

$$
\begin{aligned}
t_{0}= & t_{1}= \\
x_{0}= & x_{1}= \\
y_{0}= & y_{1}= \\
v_{0 x}= & v_{1 x}= \\
v_{0 y}= & v_{1 y}= \\
a_{x}= & a_{y}=
\end{aligned}
$$

b) Draw the velocity vector at the earlier instant and use this to determine the components of $\overrightarrow{\mathbf{v}}_{0}$. Enter these in the list above.
c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero and enter as much information about these in the list above.
d) Determine the time taken to reach the maximum height and then the horizontal distance traveled by the skier to reach her maximum height. Determine the velocity at this point.

You will now consider the motion from the highest point back to the ground.
e) Repeat the problem set-up with the"earlier" instant being that at which the skier is at maximum height and the "later" instant being the moment just before she reaches hits the ground. Determine the time taken for this portion of the motion and use it to determine the horizontal distance from the base of the ramp to the skier's landing point.

## 53 Aircraft dropping object.

An aircraft flies horizontally with a constant speed of $600 \mathrm{~km} / \mathrm{h}$ at a height of 1200 m above a flat surface. It drops an object from its underside; this object is supposed to hit a particular spot on the ground. How far (horizontally) from the spot must the aircraft be for the object to hit the spot?

## 54 Rifle shot

A rifle fires a bullet. At the moment that the bullet leaves the rifle it travels horizontally with speed $300 \mathrm{~m} / \mathrm{s}$. Determine how far the bullet will drop by the time that it reaches a vertical wall that is 100 m from the end of the rifle that is closest to the wall.

## 55 Jumping grasshopper

A grasshopper jumps, leaving the ground at an angle of $70^{\circ}$ from the horizontal. The grasshopper reaches a height of 0.60 m .
a) Determine the speed with which the grasshopper leaves the ground.
b) Determine how far the grasshopper lands from where it jumped.

## 56 Jumping lemur

A lemur (a type of primate) jumps, leaving the ground at an angle of $40^{\circ}$ from the horizontal with speed $6.0 \mathrm{~m} / \mathrm{s}$.
a) Determine the maximum height reached by the lemur.
b) Determine the amount of time for which the lemur is airborne.
c) Determine the horizontal distance traveled by the lemur from the moment it leaves the ground until it returns to the ground.

## Dynamics

## 57 Hockey pucks sliding horizontally

Three identical hockey pucks slide horizontally across a frictionless sheet of ice and they maintain the indicated speeds during a particular period. Let $F_{A}$ be the magni-
 tude of the force acting on A during this period, $F_{B}$ be the magnitude of the force acting on B , etc, $\ldots$. Which of the following is true? Explain your answer.
i) $F_{B}=2 F_{A}$ and $F_{C}=4 F_{A}$
ii) $F_{A}=2 F_{B}$ and $F_{A}=4 F_{C}$
iii) $F_{A}=F_{B}=F_{C} \neq 0$
iv) $F_{A}=F_{B}=F_{C}=0$

## 58 Ant on a stick

An ant and a bug walk along straight wires. The graph illustrates their velocities vs. time. They have the same mass. Answer the following, giving explanations for each answer.
a) How does the net force on the ant at 2.0 s compare (same, larger, four times as large,etc, ) to the net force on the ant at 4.0 s ?
b) How does the net force on the ant compare (same, always larger, sometimes
 larger, always smaller,...) to the net force on the bug?
c) Is the net force exerted on the ant ever the same as that exerted on the bug? If so, explain when.

## 59 Moving carts

Three identical carts move horizontally along tracks. Their speeds at two instants 5.0 s apart are indicated. Let $F_{A}$ be the magnitude of the net force acting on A during this interval, $F_{B}$ be the magnitude of the net force acting on B , etc, $\ldots$. Which of the following is true? Explain your answer.
i) $F_{A}>F_{B}>F_{C}$.
ii) $F_{B}=F_{C}>F_{A}$.
iii) $F_{B}=F_{C}<F_{A}$.

iv) $F_{A}=F_{B}=F_{C} \neq 0$

## 60 Riding a bicycle

Suppose that you ride a bicycle at constant speed in straight lines across various surfaces. In each of teh following, explain your answers.
a) If you ride at a constant speed along a flat horizontal surface, which of the following is true?
i) The net force on the bicycle is zero.
ii) The net force on the bicycle is in the direction of motion.
iii) The net force on the bicycle is opposite to the direction of motion.
iv) Whether the net force is zero or not depends on the speed.
b) If you ride at a constant speed along a slanted flat uphill surface, which of the following is true?
i) The net force on the bicycle is zero.
ii) The net force on the bicycle is in the direction of motion.
iii) The net force on the bicycle is opposite to the direction of motion.
iv) Whether the net force is zero or not depends on the speed.

## 61 Pushing carts

Zog and Geraldine (his wife) each push a cart along a horizontal surface where friction is negligible. Both carts are initially at rest. Zog takes the cart with mass 25 kg and exerts a force of 400 N on it for a period of 4.0 s and he then collapses and stops pushing. Geraldine has to push a cart of mass 50 kg and she is also able to exert a force of 400 N on it. Geraldine claims that it is possible for the speed of her cart to eventually reach the speed of Zog's cart. Is this true? Explain your answer.

## 62 Net force vector

Two forces, $\overrightarrow{\mathbf{F}}_{1}$ and $\overrightarrow{\mathbf{F}}_{2}$ act on an object and are as illustrated. Their magnitudes are $F_{1}=400 \mathrm{~N}$ and $F_{2}=500 \mathrm{~N}$.
a) Determine the $x$ and $y$ components of the net force.
b) Determine the magnitude of the net force acting on the object.


## 63 Net force vector

Three forces, $\overrightarrow{\mathbf{F}}_{1}, \overrightarrow{\mathbf{F}}_{2}$ and $\overrightarrow{\mathbf{F}}_{3}$ act on an object and are as illustrated. Their magnitudes are $F_{1}=$ $40 \mathrm{~N}, F_{2}=25 \mathrm{~N}$ and $F_{3}=15 \mathrm{~N}$.
a) Determine the $x$ and $y$ components of the net force.
b) Determine the magnitude of the net force acting on the object.


## 64 Forces and motion

A sleepy 30 kg dog lies on a horizontal surface. The dog is initially at rest. Subsequently a child pushes horizontally on the dog with a 150 N force. There is no friction between the dog and the surface and forces in the vertical direction are irrelevant to its motion.
a) Determine how far the dog moves after the child has pushed for 3.0 s .
b) Determine the speed of the dog after the child has pushed for 3.0 s .
c) If the child continues pushing for twice as long will the speed be twice as much? Will the distance traveled be twice as much? Explain your answers.

## 65 Horizontal motion and forces

An cart can slide horizontally left or right.
a) Suppose that the cart slides to the left with increasing speed. Which of the following is true about the net force acting on the cart? Explain your choice.
i) $\overrightarrow{\mathbf{F}}_{\text {net }}=0$
ii) $\overrightarrow{\mathbf{F}}_{\text {net }} \neq 0$ and points right.
iii) $\overrightarrow{\mathbf{F}}_{\text {net }} \neq 0$ and points left.
b) Suppose that the cart slides to the left with decreasing speed. Which of the following is true about the net force acting on the cart? Explain your choice.
i) $\overrightarrow{\mathbf{F}}_{\text {net }}=0$
ii) $\overrightarrow{\mathbf{F}}_{\text {net }} \neq 0$ and points right.
iii) $\overrightarrow{\mathbf{F}}_{\text {net }} \neq 0$ and points left.

## 66 Ice skaters

Two ice skaters, Alice with mass 55 kg and Bob with mass 85 kg are initially at rest on a frictionless horizontal surface. Bob gently pushes on Alice and while he does this she has a constant acceleration of $0.020 \mathrm{~m} / \mathrm{s}^{2}$.
a) Determine the force exerted by Bob on Alice while she accelerates.
b) Determine the force exerted by Alice on Bob while Alice accelerates.
c) Determine the acceleration of Bob during the period while he is pushing on Alice.

## 67 Suspended object in equilibrium

A 2.50 kg ring is suspended from the ceiling and is held at rest by two ropes as illustrated. Rope 2 pulls horizontally. The aim of this exercise is to use Newton's $2^{\text {nd }}$ Law to determine the tension in each rope. One piece of background information that you will need to answer this is that the magnitude of the grav-
 itational force on an object of mass $m$ is $F_{g}=m g$.
a) Draw a free body diagram for the ring. Label the tension forces $\overrightarrow{\mathbf{T}}_{1}$ and $\overrightarrow{\mathbf{T}}_{2}$.
b) Write Newton's $2^{\text {nd }}$ Law in its component form, i.e. write

$$
\begin{align*}
F_{\text {net } x} & =\Sigma F_{i x}=m a_{x}  \tag{1}\\
F_{\text {net } y} & =\Sigma F_{i y}=m a_{y} \tag{2}
\end{align*}
$$

Insert as much information as possible about the acceleration. You will return to these equations shortly; they will generate the algebra that eventually gives you the tensions.
c) These equations require all components of all forces, including the two unknown tension forces. In order to manage these, you should express the components of each tension force in terms of its magnitude. When doing this denote the magnitude of the tension in rope 1 by $T_{1}$ and for rope 2 , by $T_{2}$.
d) List as much information as possible about each component for each force; each could be a number or an algebraic expression. Use one of the two formats below.

$$
\begin{aligned}
w_{x} & =\cdots \\
w_{y} & =\cdots \\
T_{1 x} & =\cdots \\
T_{1 y} & =\cdots
\end{aligned}
$$

| Force | $x$ comp | $y$ comp |
| :---: | :---: | :---: |
| $\overrightarrow{\mathbf{w}}$ |  |  |
| $\overrightarrow{\mathbf{T}}_{1}$ |  |  |
| $\vdots$ |  |  |

e) Use Eq. (1) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (2). You should get two expressions that contain the two unknowns $T_{1}$ and $T_{2}$. Solve them for the unknowns.
f) If you had one rope that is rated to break when the tension exceeds 30 N and another rated to break when the tension exceeds 40 N , which one would you use to suspend the object as illustrated above?

## 68 Object suspended at rest

A 0.20 kg block is suspended by two ropes as illustrated. The tension in the left rope is $T_{1}$ and in the right rope it is $T_{2}$.
a) Is $T_{1}$ the same as or different to $T_{2}$ ? Explain your answer.
b) Determine the magnitude of the tension in each rope.


## 69 Stretched clothesline

A clothesline (rope) initially droops approximately horizontally between two poles. A hanger is suspended at rest from the midpoint of the clothesline.
a) Is it possible to stretch the clothes line so that it is perfectly horizontally while the hanger is suspended? Explain your answer using Newton's laws.
b) The hanger is removed. Is it possible to stretch the clothes line so that it is perfectly horizontally? Explain your answer using Newton's laws.

## 70 Multiple forces and motion

A 20.0 kg box can move along a frictionless horizontal surface. Three people pull horizontally on the box with the indicated forces.

a) Draw a free body diagram for the box.
b) Write Newton's Second Law in its component form, i.e. write

$$
\begin{aligned}
F_{\text {net } x} & =\Sigma F_{x}=\cdots \\
F_{\text {net } y} & =\Sigma F_{y}=\cdots
\end{aligned}
$$

Insert as much information as possible about the components of acceleration at this stage. You will return to these equations soon; they will generate the algebra that eventually gives you the acceleration and the normal force.
c) List all the components of all the forces, using one of the two formats below.

$$
\begin{aligned}
F_{1 x} & =\cdots \\
F_{1 y} & =\cdots \\
F_{2 x} & =\cdots \\
F_{2 y} & =\cdots \\
F_{3 x} & =\cdots \\
F_{3 y} & =\cdots
\end{aligned}
$$

| Force | $x$ comp | $y$ comp |
| :---: | :---: | :---: |
| $\overrightarrow{\mathbf{F}}_{1}$ |  |  |
| $\overrightarrow{\mathbf{F}}_{2}$ |  |  |
| $\overrightarrow{\mathbf{F}}_{3}$ |  |  |

Use these to determine the components of the net force.
d) Use Newton's second law, which you wrote earlier, to determine the components of the acceleration of the box.
e) If the box were moving right while these forces act would it slow down or speed up?
f) If the box were moving left while these forces act would it slow down or speed up?

## 71 Dynamics of a single object

A 5.0 kg box can move along a frictionless horizontal surface. A person exerts a force at the illustrated angle. The aim of this exercise is to use Newton's laws to determine the acceleration of the box and the normal force exerted by the surface (provided that
 the box stays on the surface).

The entire collection of these steps is called "applying Newton's laws of mechanics to determine the acceleration of the block."
a) Draw a free body diagram for the block.
b) Write Newton's Second Law in its component form, i.e. write

$$
\begin{align*}
F_{\text {net } x} & =\Sigma F_{x}=\cdots  \tag{3}\\
F_{\text {net } y} & =\Sigma F_{y}=\cdots \tag{4}
\end{align*}
$$

Insert as much information as possible about the components of acceleration at this stage. You will return to these equations shortly; they will generate the algebra that eventually gives you the acceleration and the normal force.
c) Determine the magnitude of the gravitational force. Let $n$ be the magnitude of the normal force. Do you think that $n=m g$ ?
d) List all the components of all the forces, using one of the two formats below.

$$
\begin{aligned}
w_{x} & =\cdots \\
w_{y} & =\cdots \\
n_{x} & =\cdots \\
n_{y} & =\cdots \\
\vdots &
\end{aligned}
$$

| Force | $x$ comp | $y$ comp |
| :---: | :---: | :---: |
| $\overrightarrow{\mathbf{w}}$ |  |  |
| $\overrightarrow{\mathbf{n}}$ |  |  |
| $\vdots$ |  |  |

e) Use Eq. (3) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (4). Solve these for the acceleration and the magnitude of the normal force. Is $n=m g$ ?
f) Suppose that rather than pull up, the person pushed down on the box at the same angle from the left and with the same force. Would the acceleration and normal forces differ from the case where the person pulled up?
g) You may have noticed that the acceleration does not depend on the normal force. This is only true if there is no friction. It turns out that when friction is present, the magnitude of friction force increases as the normal force increases. Knowing this, would pulling up or pushing down give a larger acceleration?

## 72 Box on the floor of an elevator

A 20 kg box sits on the floor of an elevator.
a) During an initial 1.5 s period the elevator moves upward with a speed that increases steadily from $0 \mathrm{~m} / \mathrm{s}$ to $7.5 \mathrm{~m} / \mathrm{s}$. Determine the normal force exerted by the floor on the box during this period.
b) During an later 2.5 s period the elevator moves upward with a speed that decreases steadily from $7.5 \mathrm{~m} / \mathrm{s}$ to $0 \mathrm{~m} / \mathrm{s}$. Determine the normal force exerted by the floor on the box during this period.

## 73 Free fall in an elevator

A phone of mass $m$ sits on the floor of an elevator, which is initially at rest. The elevator cable snaps and the elevator and phone then undergo free fall. While they do this which is true of the magnitude of the normal force, $n$, acting on the phone? Explain your choice.
i) $n=0$.
ii) $m g>n>0$.
iii) $n=m g$.
iv) $n>m g$

## 74 Normal forces

Two identical boxes are at rest on a rough horizontal surface. A person pulls on each with the same force but in different directions on the boxes. On box A it pulls vertically and on box $B$ it pulls at an angle of $45^{\circ}$ from the vertical. Which of the following is true regarding the normal force exerted on A
 and that exerted on B? Explain your choice.
i) $n_{\mathrm{A}}=n_{\mathrm{B}}$.
ii) $n_{\mathrm{A}}<n_{\mathrm{B}}$.
iii) $n_{\mathrm{A}}>n_{\mathrm{B}}$.

## 75 Accelerating box

A rope pulls on a 30 kg which slides across a frictionless horizontal surface. The box moves along the surface to the right with acceleration $3.5 \mathrm{~m} / \mathrm{s}^{2}$.
a) Determine the tension in the rope.

b) Determine the normal force exerted by the floor on the box.

## 76 Forces and motion

Various forces act on a box with mass 80 kg that can slide along a frictionless horizontal surface.
a) Determine the acceleration of the box.

b) Suppose that at one moment the box was moving left. Will it slow down, speed up or move with constant velocity in the next moments? Explain your answer
c) Suppose that at one moment the box was moving right. Will it slow down, speed up or move with constant velocity in the next moments? Explain your answer.

## 77 Multiple forces on a single object

A 25.0 kg crate can move along a frictionless horizontal surface. Two people exert forces on the crate as illustrated.

a) Determine the acceleration of the crate.
b) Determine the normal force on the crate.

## 78 Elevator

Alice, with mass 58 kg stands in an elevator, which moves vertically. The elevator accelerates (at a constant rate) from rest to speed of $4.0 \mathrm{~m} / \mathrm{s}$ during a period of 1.5 s . It then moves with a constant velocity for a period of 10.0 s. Finally it slows to a stop at a constant rate during a period of 2.0 s .
a) Determine the total distance traveled by Alice during the entire journey.
b) Determine the normal force exerted by the elevator floor on Alice during each of the three stages of motion.

## 79 Bathroom scale in a train

A bathroom scaled reads the normal force that it exerts on the person standing on the scale. Suppose that a bathroom scale is fixed to the floor of a train and a person stands on the scale. The train travels at a constant speed along horizontal sections of track and then up and down inclined sections (the slopes of the inclined sections are the same). In the following explain your answers.
a) Is the reading on the scale when traveling up the inclined section the same as, smaller than or larger than when traveling along the horizontal section?
b) Is the reading on the scale when traveling down the inclined section the same as, smaller than or larger than when traveling up the inclined section?
c) Would the reading on the scale while the train travels up the inclined section depend on the speed with which it travels?

## 80 Blocks on ramps

Two blocks with the same mass are at rest on the illustrated inclined surfaces. Which of the following is true about the magnitudes of the normal forces that act on the blocks? Explain your answer.

i) $n_{A}>n_{B}$.
ii) $n_{\mathrm{A}}=n_{\mathrm{B}}$.
iii) $\frac{1}{2} n_{\mathrm{B}}<n_{\mathrm{A}}<n_{\mathrm{B}}$.

iv) $n_{\mathrm{A}}=\frac{1}{2} n_{\mathrm{B}}$.
v) $n_{\mathrm{A}}<\frac{1}{2} n_{\mathrm{B}}$.

## 81 Different boxes on the same ramp

Two boxes, one red (mass 3.0 kg ) and the other blue (mass 6.0 kg ) are each on frictionless ramps inclined at an angle of $10^{\circ}$ from the horizontal. They are released from rest. Let $a_{\text {red }}$ be the acceleration of the red box and $a_{\text {blue }}$ that of the blue box. Which of the following is true? Explain your answer.
i) $a_{\text {blue }}=\frac{1}{2} a_{\text {red }}$
ii) $a_{\text {blue }}=a_{\text {red }}$
iii) $a_{\text {blue }}=2 a_{\text {red }}$
iv) $a_{\text {blue }}>2 a_{\text {red }}$

## 82 Snowboarder sliding up a ramp

An 80 kg snowboarder slides along a horizontal surface with a speed of $25 \mathrm{~m} / \mathrm{s}$ and approaches a surface inclined at $10^{\circ}$ above the horizontal. Both surfaces are frictionless and you can ignore air resistance.
a) Determine the acceleration of the snowboarder along the inclined surface.
b) Determine how far along the inclined surface the snowboarder slides before stopping.
c) Determine the maximum vertical height ascended by the snowboarder along the ramp.

## 83 Pushing a box at constant speed

A 10 kg box can move along a rough horizontal surface. You discover that when you push horizontally with a force of 50 N that the box moves with constant speed. Which of the following is true about the magnitude of the friction force? Explain your answer.
i) $f=10 \mathrm{~N}$
ii) $f=50 \mathrm{~N}$
iii) $f=98 \mathrm{~N}$
iv) $f$ is more than 50 N but less than 98 N .
v) $f$ is more than 98 N .

## 84 Crate in a truck

A crate lies on the flat, horizontal bed of a truck. The truck moves along a horizontal surface. In the following the truck moves so that the crate does not slip along the bed of the truck.
a) Is there a friction force between the crate and the truck when they both move north at the same constant speed along a straight horizontal road? Explain your answer.
b) Suppose that the truck speeds up while heading North. In which direction does the friction force point when it does this? Explain your answer.
c) Suppose that the truck slows down while heading North. In which direction does the friction force point when it does this? Explain your answer.

Explain your answers. Hint: Consider applying Newton's second law to the crate.

## 85 Dynamics of a single object with friction

A 15.0 kg box moves rightward along a horizontal surface. A rope pulls with a force at the illustrated angle. The coefficient of kinetic friction is 0.250 . The primary aim of this exercise will be to determine the acceleration of the box.

a) Draw a free body diagram for the block.
b) Write Newton's Second Law in its component form, i.e. write

$$
\begin{align*}
F_{\text {net } x} & =\Sigma F_{x}=\cdots  \tag{5}\\
F_{\text {net } y} & =\Sigma F_{y}=\cdots \tag{6}
\end{align*}
$$

Insert as much information as possible about the components of acceleration at this stage. Can you describe in words what these equations are telling you to do?
c) Determine the magnitude of the gravitational force. Let $n$ be the magnitude of the normal force. Using this write an expression for the magnitude of the friction force. Do you know the exact number for the friction force at this point?
d) List all the components of all the forces, using one of the two formats below.

$$
\begin{aligned}
w_{x} & =\cdots \\
w_{y} & =\cdots \\
n_{x} & =\cdots \\
n_{y} & =\cdots
\end{aligned}
$$

| Force | $x$ comp | $y$ comp |
| :---: | :---: | :---: |
| $\overrightarrow{\mathbf{w}}$ |  |  |
| $\overrightarrow{\mathbf{n}}$ |  |  |
| $\vdots$ |  |  |

e) Use Eq. (5) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (6). Does either give the acceleration immediately? Can one of then at least give the normal force immediately?
f) Determine the normal force and use this result to find the acceleration.
g) What tension would be required for the box to have acceleration $4.00 \mathrm{~m} / \mathrm{s}^{2}$ to the right?

## 86 Box on a ramp with friction

A 10 kg box can move along a 4.0 m long rough ramp angled $20^{\circ}$ from the horizontal. The coefficient of kinetic friction between the box and the ramp is 0.25 and the coefficient of static friction is 0.30 . The box is released from rest at the top of the ramp and moves down the ramp. The aim of this exercise is to determine the speed of the box at the bottom of the
 ramp.
a) Draw a free body diagram for the box.
b) Describe your choice of $x$ and $y$ axes.
c) Write Newton's Second Law in vector form and also in its component form, i.e. write

$$
\begin{align*}
F_{\text {net } x} & =\Sigma F_{x}=\cdots  \tag{7}\\
F_{\text {net } y} & =\Sigma F_{y}=\cdots \tag{8}
\end{align*}
$$

where $x$ and $y$ refer to your specially chosen axes. Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.
d) Determine expressions for the magnitudes of the gravitational and the friction forces.
e) List all the components of all the forces, using one of the two formats below.

$$
\begin{aligned}
F_{\mathrm{g} x} & =\cdots \\
F_{\mathrm{g} y} & =\cdots \\
n_{x} & =\cdots \\
n_{y} & =\cdots
\end{aligned}
$$

| Force | $x$ comp | $y$ comp |
| :---: | :---: | :---: |
| $\overrightarrow{\mathbf{F}}_{\mathrm{g}}$ |  |  |
| $\overrightarrow{\mathbf{n}}$ |  |  |
| $\vdots$ |  |  |

f) Use Eq. (7) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (8). Use the resulting equations to find the acceleration of the box.
g) Determine the speed of the box when it reaches the bottom of the ramp.
h) Do these results depend on the mass of the box?
i) What would be the minimum force, pushing parallel to the ramp, required to keep the box at rest?

## 87 Box on a ramp with friction

A box can move along a rough ramp which makes an angle $\theta$ from the horizontal and which has length $L$. The box is launched with speed $v$ from the top of the ramp. The aim of this exercise is to determine the coefficient of friction needed to bring the box to
 a stop at the bottom of the ramp.
a) The first part of the solution uses kinematics to assess the acceleration of the box. Using kinematics, and eventually dynamics, is greatly simplified by choosing an appropriate " $x$ " and a " $y$ " axis. These do not have to be along the usual vertical and horizontal directions. Regardless of the axes that you choose, the usual general kinematics and dynamics equations will be valid. Describe the " $x$ " and " $y$ " axes that you will use.
b) Determine an expression for the magnitude of the acceleration of the box, $a$, in terms of variables relevant to this problem, such as $L, \theta, v$, and possibly the mass of the box, $m$.
c) Draw a free body diagram for the box.
d) Write Newton's Second Law in vector form and also in its component form, i.e. write

$$
\begin{align*}
F_{\text {net } x} & =\Sigma F_{i x}=\cdots  \tag{9}\\
F_{\text {net } y} & =\Sigma F_{i y}=\cdots \tag{10}
\end{align*}
$$

where $x$ and $y$ refer to your specially chosen axes. Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.
e) Determine expressions for the magnitudes of the gravitational and the friction forces.
f) List all the components of all the forces, using one of the two formats below.

$$
\begin{aligned}
F_{g x} & =\cdots \\
F_{g y} & =\cdots \\
n_{x} & =\cdots \\
n_{y} & =\cdots
\end{aligned}
$$

$$
\vdots
$$

| Force | $x$ comp | $y$ comp |
| :---: | :---: | :---: |
| $\overrightarrow{\mathbf{F}}_{g}$ |  |  |
| $\overrightarrow{\mathbf{n}}$ |  |  |
| $\vdots$ |  |  |

g) Use Eq. (9) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (10). Use the resulting equations to find an expression for the coefficient of kinetic friction. Does the result depend on the mass of the box?
h) Suppose that the ramp is a roof whose length is 5.0 m and which is angled at $15^{\circ}$ from the horizontal. If the box is pushed with speed $4.0 \mathrm{~m} / \mathrm{s}$, determine the coefficient of friction needed to stop the box at the bottom of the roof.

## 88 Box pushed up a ramp with friction

A block of mass $m$ slides up a ramp at angle $\theta$ from the horizontal. The coefficient of kinetic friction between the surfaces is $\mu_{\mathrm{k}}$. A person pushes horizontally with a force $F_{\mathrm{p}}$. The object of this exercise is to determine an expression for the acceleration of the block
$a=$ formula involving $m, \theta, F_{\mathrm{p}}, \mu_{\mathrm{k}}, g$ and constants.


The entire collection of these steps is called "applying Newton's laws of mechanics to determine the acceleration of the block."
a) Draw a free body diagram for the block.
b) Describe the $x$ and $y$ axes that you will use.
c) Write Newton's Second Law in vector form and also in its component form, i.e. write

$$
\begin{align*}
F_{\text {net } x} & =\cdots  \tag{11}\\
F_{\text {net } y} & =\cdots \tag{12}
\end{align*}
$$

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.
d) Determine expressions for the magnitudes of all the friction and gravitational forces. Do you think that $n=m g$ ?
e) List all the components of all the forces, using one of the two formats below.

$$
\begin{aligned}
F_{g x} & =\cdots \\
F_{g y} & =\cdots \\
n_{x} & =\cdots \\
n_{y} & =\cdots \\
\vdots &
\end{aligned}
$$

| Force | $x$ comp | $y$ comp |
| :---: | :---: | :---: |
| $\overrightarrow{\mathbf{n}}$ |  |  |
| $\vdots$ |  |  |
| $\vdots$ |  |  |

f) Note that $F_{\text {net } x}=\sum_{i} F_{i x}$ (i.e. the $x$ component of the net force is the sum of the $x$ components of individual forces). Use this and Eq. (11) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (12). Use the resulting algebraic expressions to get an expression for $n$. Is this $m g$ ? Use the resulting equations to get an expression for $a$.

## 89 Brick on a slope

A 1.50 kg brick is at rest on an inclined surface. The angle between the surface and the horizontal is $10^{\circ}$.
a) Determine the friction force that the surface exerts on the brick.
b) Determine the minimum coefficient of static friction between these surfaces that allows for the brick to stay at rest.

## 90 Sledding on a slope

A person in a sled is at rest at the top of a slope that is angled $15^{\circ}$ above the horizontal. The length of the slope is an 800 m (about 0.5 mi ). The combined mass of the person and sled is 90 kg . They are released from the top of the slope and slide straight down without pushing. While this happens the air exerts a constant force of 150 N exactly opposite to the direction in which they move.
a) Determine the acceleration of the person and sled. You must solve this by starting with a FBD, using Newton's second law, finding components,.... Simply looking up a formula is not adequate.
b) Determine the time taken for the person and sled to reach the bottom of the slope.
c) Determine the speed of the person and sled at the bottom of the slope.

## 91 Stacked objects on a surface

Various boxes are arranged as illustrated. The 4 kg block lies on a table with a horizontal surface. There is friction between the block and the table.
a) Suppose that all the blocks are at rest. Is there a friction force between the 4 kg block and the table? If so, in which direction? Explain your answer.

b) Suppose that all the blocks are at rest. Is there a friction force between the 2 kg block and the 4 kg block? If so, in which direction? Explain your answer.
c) Suppose that the block on the table accelerates to the right and the 2 kg block does not slip relative to the 4 kg block. Is there a friction force between the 2 kg block and the 4 kg block? If so, in which direction? Explain your answer.

92 Dynamics of connected objects; level/suspended blocks without friction
Two blocks are connected by a string, which runs over a massless pulley. A 10 kg block is suspended and a 5.0 kg block can slide along a frictionless horizontal surface. The string connected to the block on the surface runs horizontally. The blocks held at rest and then released. They move, constantly speeding up. Which of the following is true regarding the tension
 in the connecting string, $T$, while they move? Explain your choice.
i) $T=0$.
ii) $98 \mathrm{~N}>T>0$.
iii) $T=98 \mathrm{~N}$.
iv) $T>98 \mathrm{~N}$

## 93 Bug walking in a circle

A bug walks at a constant speed in a circular path on a horizontal surface. Which vector best illustrates the net force on the bug at the illustrated moment? Explain your choice.

## 94 Connected balls swinging in horizontal circles

Two balls are connected by strings. They are on a horizontal frictionless surface and swing in horizontal circles about a central point; both strings point radially outward from the center as the balls move. The diagram illustrates this as viewed from above.
a) Draw a free body diagram, as viewed from the side, for ball B at the moment when the balls are at the leftmost location of their motion.

) Draw the direction of the net force on the ball B at the moment when the balls are at the leftmost location of their motion.
c) How does the tension in the inner string compare (larger, smaller, same) to that in the string connecting the two balls? Explain your answer.

## 95 Connected balls swinging in horizontal circles

Two balls are connected by strings. They are on a horizontal frictionless surface and swing in horizontal circles about a central point; both strings point radially outward from the center as the balls move. The diagram illustrates this as viewed from above.
a) Suppose that A has a larger mass than B. Is the tension in the inner string the same as, larger than or equal to the tension in the outer string? Explain your answer.
b) Suppose that A has a smaller mass than B. Is the tension in the inner string the same as, larger than or equal to the tension in the outer string? Explain your answer.
c) In which arrangement (larger mass on the outside, smaller mass on the outside) will the tension in the outer string be larger? Assume that both arrangements swing with the same speed. Explain your answer.

## 96 Car turning

A 3000 kg car travels along a flat (horizontal) road. The road curves with a radius of 350 m . The maximum speed with which the car can do the turn is $18 \mathrm{~m} / \mathrm{s}$.
a) Determine the net force on the car. What force provides the net force?
b) Determine the (minimum) coefficient of static friction that allows the car to follow this curve at the given speed.

## 97 Ball swinging in a vertical circle

A 0.20 kg ball swings with in a vertical circle at the end of a string of length 0.50 m .
a) Draw a free body diagram for the ball at the highest point of the circle. Draw a free body diagram at the lowest point.
b) Suppose that the speed of the ball is constant throughout its motion. How does net force at the
 highest point of the circle compare (larger, smaller, same) to that at the lowest point of the circle? Use you answer to compare (larger, smaller, same) the tension in the string at the lowest point of the circle to the tension at the highest point of the circle.
c) Suppose that the string will break if the tension in it exceeds 5.0 N . Use Newton's second law to analyze the situation where the tension is largest (i.e. highest or lowest point) and determine the maximum speed with which the ball can move so that the string does not break.
d) In general the speed of the ball can vary as it swings. As the speed decreases does the tension at the top of the circle increase, decrease or stay constant? Determine the minimum speed so that the tension is not zero. Describe what happens if the speed drops beneath this.

## 98 Monkey on a rope

A monkey with mass $m_{\text {monkey }}$ swings from a rope. Is the tension in the rope at the bottom of the swing larger than, smaller than or the same as $m_{\text {monkey }} g$ ? Explain your answer.

## 99 Sliding inside a hoop

A small 0.10 kg object slides around the inside of a vertical hoop with radius 0.25 m .
a) The speed of the object at the top of the hoop is $2.0 \mathrm{~m} / \mathrm{s}$. Determine the normal force exerted by the hoop on the object at the top of the loop.
b) The object reaches the bottom of the hoop with speed $3.71 \mathrm{~m} / \mathrm{s}$. Determine the normal force exerted by the hoop on the object at the bottom of the loop.

c) Determine the minimum speed at the top of the loop so that the object stays on the inside surface of the loop.

## 100 Block inside a revolving drum

A spinning drum has vertical wooden sides. A wooden block block is placed inside the drum and the drum is eventually made to spin with a constant angular velocity. This is done is such a way that the block does not slide relative to the drum; it rotates at the same rate as the drum. In the first part of the problem suppose that the mass of the block is 2.0 kg , the coefficient of static friction is 0.60 and the radius of the drum is 0.80 m .

a) Draw a free body diagram for the block, identify the direction of the acceleration and write Newton's 2nd law in component form. Is $n=m g$ in this case?
b) Determine an expression for the minimum angular velocity with which the drum must rotate so that the block does not slip.

Now consider the more general case where the drum has radius $r$ and the coefficient of static friction is $\mu_{s}$.
c) Determine an expression for the minimum angular velocity (in terms of $g, r$ and $\mu_{s}$ ) so that the block does not slip. Does it depend on the mass of the block?

Now suppose that the spinning drum has tilted sides. A block is placed inside the drum and the drum is eventually made to spin with a constant angular velocity $\omega$. This is done is such a way that the block does not slide relative to the drum; it rotates at the same rate as the drum. Let $r$ be the distance from the block to the axle of the drum.
d) Determine the minimum angular velocity re-
 quired for the block not to slip when $\theta=75^{\circ}$, $r=2.5 \mathrm{~m}$ and the coefficient of static friction is 0.2 .

## 101 Banked turn on a road

A road is constructed with a turn of radius 300 m . The surface of the road is banked at an angle in order to assist cars to make the turn. A car travels through this turn with constant speed of $35 \mathrm{~m} / \mathrm{s}$ (about 80 mph ).
a) Suppose that there is no friction between the tires and road. Determine the angle at which the road must be banked so that the car can complete the turn without slipping.
b) There usually is friction between the tires and road. For wet concrete and rubber the coefficient of static friction is 0.30 and the coefficient of kinetic friction is 0.20 . Determine the minimum angle at which the road must be banked so that the car completes the turn without slipping.

## Gravitation

## 102 Moons of planet

A planet has two moons. Moon B has twice the mass of Moon A. Moon B has an orbit with twice the radius of the orbit of Moon A.
a) Let $F_{A}$ be the force exerted by the planet on Moon A and $F_{B}$ be the force exerted by the planet on Moon B. Which of the following is true? Explain your answer.
i) $F_{A}=\frac{1}{4} F_{B}$
ii) $F_{A}=\frac{1}{2} F_{B}$
iii) $F_{A}=F_{B}$
iv) $F_{A}=2 F_{B}$
v) $F_{A}=4 F_{B}$
b) Let $a_{A}$ be the acceleration of Moon A and $a_{B}$ be the acceleration of Moon B. Which of the following is true? Explain your answer.
i) $a_{A}=\frac{1}{4} a_{B}$
ii) $a_{A}=\frac{1}{2} a_{B}$
iii) $a_{A}=a_{B}$
iv) $a_{A}=2 a_{B}$
v) $a_{A}=4 a_{B}$

## 103 Mars' orbit

Mars orbits the Sun in an approximately circular orbit with radius $2.28 \times 10^{11} \mathrm{~m}$. The period of orbit is 1.88 years and Mars' mass is $6.42 \times 10^{23} \mathrm{~kg}$.
a) Use the data to determine the acceleration of Mars and the net force on Mars.
b) Use the result of the previous part and gravitational force exerted by the Sun on Mars to determine the mass of the Sun.

## Work and Energy

## 104 Work done on a crate

A 20 kg crate moves along a horizontal frictionless surface. One person pulls up on the crate from the right and another pulls up on the left as illustrated. The crate is initially at rest and subsequently slides right for 5.0 m .

a) Determine the work done by each of the following forces: gravity, normal force, and the forces exerted by the two people.
b) Determine the net work done on the crate.

## 105 Barge in a canal

A barge can move along a straight canal. The barge is pulled by a donkey that walks along the bank of the canal. A rope connects the donkey to the barge. The donkey pulls the barge in a straight line for a distance of 750 m .
a) Suppose that the rope makes an angle of $15^{\circ}$ from the forwards direction of the barge's motion and the the donkey exerts a constant force of 275 N on the barge. Determine the work done by the donkey.
b) Now suppose that the rope length is changed and the result is that the rope makes an angle of $8.0^{\circ}$ from the forwards direction of the barge's motion. Determine the force that must be exerted by the donkey so that it does the same work as it had done previously.

## 106 Raising an elevator

An elevator is raised by a cable and slows to a stop.
a) While the elevator slows is the work done by the tension in the cable positive, negative or zero? Explain your answer.
b) While the elevator slows is the magnitude of the work done by tension larger than, smaller than or the same as the magnitude of the work done by gravity? Explain your answer.

## 107 Object sliding down a ramp

A 4.0 kg box slides down a ramp that makes and angle of $20^{\circ}$ from the horizontal. The length of the ramp is 3.0 m .
a) Determine the work done by gravity as the box slides all the way down the ramp from the top to the bottom.
b) Determine the work done by the normal force as the box slides all the way down the ramp from the top to the bottom.

## 108 Cart sliding up and down a ramp

A cart approaches a ramp. The cart slides up the ramp, reverses direction and then slides down.
a) Consider the segment of the motion where the cart slides up the ramp. Is the work done by gravity positive, negative or zero? Explain your answer.
b) Consider the segment of the motion where the cart slides down the ramp. Is the work done by gravity positive, negative or zero? Explain your answer.

## 109 Kinetic energy

Two boxes on move on a frictionless horizontal surface. The speed of the box in case 2 is twice the speed of the box in case 1 . Let $K_{1}$ be the kinetic energy of the block in Case 1 and $K_{2}$ be the kinetic energy of the block in Case 2. Which of the following is true? Explain your answer.
i) $K_{1}=\frac{1}{4} K_{2}$
ii) $K_{1}=\frac{1}{2} K_{2}$
iii) $K_{1}=K_{2}$

iv) $K_{1}=2 K_{2}$
v) $K_{1}=4 K_{2}$

## 110 Molecular kinetic energy

The average kinetic energy of molecules in a gas only depends on the temperature of the gas (side note: this is about translational kinetic energy only). Consider a gas of helium molecules (each Helium molecule consists of just one Helium atom) and a gas of Neon molecules (each Neon molecule consists of one Neon atom). Suppose that these gases are both at the same temperature. For these gases how does the typical speed of the Helium molecules compare (same, smaller, larger, ...) to that of the Neon molecules.

## 111 Pulling a box: energy and speed

Two boxes on a frictionless horizontal surface are initially at rest. Both are pulled by a rope with the same tension for the same distance.
a) Let $K_{1}$ be the kinetic energy of the block in Case 1 after it has


Case 2

iii) $K_{1}=K_{2}$
iv) $K_{1}=2 K_{2}$
v) $K_{1}=4 K_{2}$
b) Let $v_{1}$ be the speed of the block in Case 1 after it has been pulled through distance $d$. Let $v_{2}$ be the speed of the block in Case 2 after it has been pulled through distance $d$. Which of the following is true? Explain your answer.
i) $v_{1}=\frac{1}{4} v 2$
ii) $v_{1}=\frac{1}{2} v_{2}$
iii) $v_{1}=v_{2}$
iv) $v_{1}=2 v_{2}$
v) $v_{1}=4 v_{2}$

## 112 Force, distance and speed

A rock is initially at rest on a horizontal sheet of ice. Alice pushes the rock with a constant horizontal force. The rock moves in a straight line, first passing Bob who is 5.0 m from the initial location and then Carol, who is 5.0 m from Bob.
a) Let $\Delta K_{1}$ be the change in the rock's kinetic energy from its initial location until it reaches Bob. Let $\Delta K_{2}$ be the change in the rock's kinetic energy from the moment it passes Bob until it reaches Carol. Which of the following is true? Explain your answer.
i) $\Delta K_{2}<\Delta K_{1}$
ii) $\Delta K_{2}=\Delta K_{1}$
iii) $\Delta K_{2}>\Delta K_{1}$
b) Let $v_{1}$ be the rock's speed as it reaches Bob. Let $v_{2}$ be the rock's speed as it reaches Carol. Which of the following is true? Explain your answer.
i) $v_{2}=v_{1}$
ii) $\sqrt{2} v_{1}>v_{2}>v_{1}$
iii) $v_{2}=\sqrt{2} v_{1}$
iv) $v_{2}=2 v_{1}$
v) $v_{2}>2 v_{1}$

## 113 Pushing a crate across a rough floor

A 10 kg crate can move across a rough horizontal floor. One person pulls up on the crate from the right and another pushes down as illustrated. The crate is initially at rest and subsequently slides right for 5.0 m . The coefficient of kinetic friction between
 the crate and floor is 0.20 .
a) Determine the friction force on the crate.
b) Determine the work done by each of the following forces: gravity, normal force, friction force and the forces exerted by the two people.
c) Determine the speed of the crate at the moment that it has moved 5.0 m from its original location.

## 114 Crate sliding down a ramp

A 6.00 kg crate is at rest at the top of a frictionless ramp that is inclined at $25^{\circ}$ from the horizontal. The length of the ramp is 3.00 m . Use work and energy to answer the following questions.
a) The crate is released. Determine the net work done on the crate as it moves from the top to the bottom of the ramp.
b) Use the net work to determine the speed of the block as it reaches the bottom of the ramp.

## 115 Sliding hockey puck

A 0.150 kg hockey puck slides along a horizontal surface. Initially the surface is frictionless and the puck slides with speed $18 \mathrm{~m} / \mathrm{s}$. The puck then hits a rough section of floor which provides a constant friction force. The puck stops a distance of 2.5 m after hitting the rough section.
a) Determine the net work done on the puck as it moves across the rough section. Hint: consider kinetic energy.
b) Use the net work to determine the magnitude of the friction force acting on the puck.
c) Determine the coefficient of kinetic friction between the puck and the floor.

## 116 Energy and Projectiles

A 15 kg cannonball is fired, leaving the ground with speed $30 \mathrm{~m} / \mathrm{s}$. We aim to determine the speed of the cannonball when it reaches a height of 25 m above the ground.
a) Sketch the situation showing the trajectory of the cannonball. Indicate an "initial" and a "final" moment.
b) Determine the total energy at the moment that the cannonball is launched.
c) Determine the total energy when the cannonball is 25 m above the ground.
d) Determine the kinetic energy of the cannonball when it is 25 m above the ground.
e) Determine the speed of the cannonball when it is 25 m above the ground.

## 117 Cart and ramp

A 20 kg cart is launched with speed $16 \mathrm{~m} / \mathrm{s}$ at the base of a frictionless ramp. Determine the maximum vertical height that the cart ascends as it moves up the ramp.

## 118 Work done by a string

A ball swings in a circle at the end of a string. Which of the following is true? Explain your answer.
i) The string does positive work on the ball whenever the ball speeds up and negative work on the ball whenever it slows down.
ii) The string does positive work on the ball regardless of its speed.
iii) The string does negative work on the ball regardless of its speed.
iv) The string does zero work on the ball regardless of its speed.

## 119 Spring forces

A 5.0 kg bag is suspended by a spring from the ceiling of an elevator. The spring constant is $4000 \mathrm{~N} / \mathrm{m}$.
a) The elevator and spring move up with a constant speed of $0.50 \mathrm{~m} / \mathrm{s}$. Determine the amount by which the spring is stretched.
b) The elevator and bag accelerate upwards with an acceleration of $3.0 \mathrm{~m} / \mathrm{s}^{2}$. Determine the amount by which the spring is stretched.

## 120 Monkey on a vertical spring

A monkey hangs from a spring which is attached to the ceiling of a building. The spring hangs vertically and the monkey bounces up and down without touching the floor.
a) As the monkey ascends toward and nears its highest point, the spring is compressed. Which of the following is true while this happens?
i) The spring does positive work, gravity does positive work.
ii) The spring does positive work, gravity does negative work.
iii) The spring does negative work, gravity does positive work.
iv) The spring does negative work, gravity does negative work.
b) As the monkey begins to descend away its highest point, the spring is still compressed. Which of the following is true while this happens?
i) The spring does positive work, gravity does positive work.
ii) The spring does positive work, gravity does negative work.
iii) The spring does negative work, gravity does positive work.
iv) The spring does negative work, gravity does negative work.

## 121 Sledding

King Zog, with mass 160 kg , and Queen Geraldine, with mass 80 kg , sled down an icy hill. They start from rest at the same point above the bottom of the hill. Ignore friction and air resistance. Which of the following is true regarding their speeds at the bottom of the hill? Explain your answer.
i) Same speeds.
ii) Geraldine's speed is twice that of Zog.
iii) Geraldine's speed is four times that of Zog.
iv) Zog's speed is larger than Geraldine's speed.

## 122 Skate park

A 60 kg skater approaches point A with speed $10 \mathrm{~m} / \mathrm{s}$. The skater slides along the illustrated track. Ignore friction and air resistance.

a) Determine the speed of the skater at point $B$.
b) Determine the maximum speed of the skater along the track.
c) Determine the minimum speed that the skater should have at A so that she reaches B.

## 123 Swinging monkey

A monkey grabs a 3.0 m rope and swings in a circular arc. The monkey reaches a maximum speed of $8.0 \mathrm{~m} / \mathrm{s}$. Determine the maximum angle reached by the monkey as it swings up.

## 124 Tarzan

Tarzan with mass 80 kg grabs a 5.0 m rope and swings in a circular arc. As Tarzan begins to swing the rope is at an angle of $15^{\circ}$ from the vertical. Determine Tarzan's maximum speed.

## 125 Spring bumper

Two walruses (named X and Y ), with the same masses, slide along horizontal sheets of ice. Each collides with a horizontal spring mounted to a wall; the springs are identical. Prior to hitting the spring, walrus X moved with speed twice that of walrus Y . The springs compress, bringing each walrus to a stop. Which of the following is true regarding the distances by which the springs compress? Explain your answer.
i) Springs compress by the same distance.
ii) X compresses spring by twice as much Y .
iii) X compresses spring by four times as much Y .
iv) X compresses spring by half as much Y .
v) X compresses spring by a quarter of what Y compresses.

## 126 Bungee jumper

A 100 kg person is attached to a bungee cord and, starting at rest, jumps off a bridge that is 120 m above a river. The bungee cord behaves like a spring and the length of the cord when it is unstretched is 100 m . The spring constant of the cord needs to be such that person stops just above the river.
a) Determine the total energy of the system at the moment that the person jumps.
b) Determine the total energy of the system at the moment that the person stops just above the river and use the result to determine the spring constant of the bungee cord.
c) Determine the maximum force that the bungee cord exerts on this person.
d) Now suppose that a person with mass 70 kg jumps from the same bridge using the same cord. Determine the maximum stretch in the spring, the height above the river at which the person reverses direction and the maximum force exerted on the person.

## 127 Sled and spring bumper

A 100 kg sled moves along the illustrated track. At point A it moves right with speed $5.0 \mathrm{~m} / \mathrm{s}$. While it moves friction and air resistance can be ignored.
a) Determine the energy of the sled at point A.

b) Determine the kinetic energy of the sled when it is at point $B$.
c) Determine the speed of the sled at point B.

The sled approaches a spring with spring constant $5000 \mathrm{~N} / \mathrm{m}$.
d) Determine the speed of the sled just before it hits the spring.
e) Determine the maximum distance by which the spring is compressed.
f) Determine the maximum force exerted by the spring as the sled compresses it.

## 128 Spring-launched sled

A 10.0 kg sled is pushed against a spring with spring constant $4000 \mathrm{~N} / \mathrm{m}$. The cart is held at rest while the spring is compressed by 0.45 m . It is released and then travels along the illustrated frictionless surface.

a) By the time that the sled passes point B it has left the spring. Determine the speed of the sled at point B.
b) Determine the speed of the sled at point A.

## 129 Cart and spring

A 40 kg cart can move along a horizontal surface. The cart is pushed against a spring, with spring constant $3000 \mathrm{~N} / \mathrm{m}$, compressing it by 0.35 m . It is released and begins to move right across a frictionless surface, eventually leaving the spring. After it has left the spring it enters section where there is friction. The coefficient of friction between the cart and the surface is 0.20 . The cart eventually stops in this section.
a) Determine the speed of the cart at the moment that it leaves the spring.
b) Determine the distance which the cart travels from the moment that it encounters the friction until it stops. Hint: use work and energy.

## 130 Bumper cart

A 200 kg bumper cart can move on a rough horizontal surface. The cart is pushed against a spring, with spring constant $6000 \mathrm{~N} / \mathrm{m}$, compressing it by 0.25 m . It is released and begins to move right. The coefficient of friction between the cart and the surface is 0.20 .
a) Determine the speed of the cart at the moment that it leaves the spring.
b) Suppose that while the cart moves right it is also pulled with a rope that pulls with tension 400 N to the right. Determine the speed of the cart when it leaves the spring.

## 131 Power delivered by engines

Two engines pull identical objects along horizontal surfaces. Engine A delivers 2000 W of power and engine B delivers 4000 W . Engine A pulls for 5 min and engine B for 2 min . Which of the following is true? Explain your answer.
i) Engine A delivers more work than engine B.
ii) Engine A delivers less work than engine B.
iii) Engine A delivers the same work as engine B.
iv) There is not enough information to decide.

## 132 Hoisting fish

King Zog and Queen Geraldine are fishing from a bridge and they catch identical twin fish, each with mass 5.0 kg . They hoist the fish at constant speeds to the bridge 8.0 m about the water. Zog takes 10 s to hoist his fish and Geraldine 7.5 s to hoist her fish.
a) Which of the following is true? Explain your answer.
i) Zog and Geraldine do the same work.
ii) Zog does more work than Geraldine.
iii) Zog does less work than Geraldine.
b) Which of the following is true? Explain your answer.
i) Zog and Geraldine expend the same power.
ii) Zog expends less power.
iii) Zog expends more power.

## 133 Raising an elevator

An 2000 kg elevator is lifted by a cable attached to a motor. It moves at a constant speed through a vertical distance of 25 m . It takes 75 s to do this.
a) Determine the work done by gravity as the elevator is lifted.
b) Determine the work done by the cable as the elevator is lifted.
c) Determine the power delivered by the motor as it raises the elevator.
d) Suppose that the elevator were raised so that the speed slowly decreased as it ascends through the same distance. How would the work done by the cable compare (larger, smaller,...) to that done by gravity? Explain your answer using physics. What would this imply for the power delivered by the motor compared to the situation where the elevator is raised at a constant speed?

## 134 Solar energy

The Sun delivers energy in the form of electromagnetic waves to Earth. At the outer edge of Earth's atmosphere, the total power per square meter (the solar irradiance) is $1360 \mathrm{~W} / \mathrm{m}^{2}$. This is diminished at Earth's surface by the atmosphere and depends on the angle of the Sun. At some location it is reduced to $1050 \mathrm{~W} / \mathrm{m}^{2}$.
a) A satellite uses solar panels to gather energy. If the satellite requires 10 kW of power, what would the necessary area of its solar panels be (assuming that the panels are $100 \%$ efficient)?
b) Suppose that a solar plant on Earth is required to provide $5.0 \times 10^{9} \mathrm{~J}$ of energy per hour. Determine the area of the Earth's surface that the solar panels would have to cover if they are only $25 \%$ efficient.

## Momentum

## 135 Colliding balls

Two balls are initially as illustrated. The 5.0 kg ball moves directly toward the 10 kg ball. The balls collide and subsequently the 10 kg ball moves right with speed $3.0 \mathrm{~m} / \mathrm{s}$. The aim of this exercise will be to determine the speed and direction of motion of the smaller ball after the collision. The
 balls are isolated from all other objects.
a) Determine the momentum of the smaller ball before the collision.
b) Determine the momentum of the larger ball before the collision.
c) Determine the momentum of the system before the collision.
d) Determine the momentum of the larger ball after the collision.
e) Determine the momentum of the smaller ball after the collision.
f) Determine the velocity of the smaller ball after the collision.
g) Suppose that the construction of the balls was different and that after the collision the larger ball moves right with speed $5.0 \mathrm{~m} / \mathrm{s}$. Determine the velocity of the smaller ball after the collision.

## 136 Colliding balls

Two balls initially move toward each other as illustrated. The 3.0 kg ball moves directly toward the 6.0 kg ball. The balls collide and subsequently the 3.0 kg ball moves left with speed $4.0 \mathrm{~m} / \mathrm{s}$. The balls are isolated from all other objects. Determine the velocity of the larger ball after the collision.


## 137 Space collision

A 100 kg astronaut is at rest in space. A 0.0050 kg fleck of paint moves toward the astronaut with speed $15 \times 10^{3} \mathrm{~m} / \mathrm{s}$. It collides with a sticks to the astronaut.
a) Write an expression for the total momentum of the system before the collision in terms of the masses and speeds of the astronaut and paint. Determine the total momentum of the system before the collision (assume that the paint fleck initially moves along the positive $x$ axis).
b) Write and expression for the total momentum of the system after the collision in terms of the masses and speeds (after collision) of the astronaut and paint.
c) Use momentum conservation to determine the speed of the astronaut after the collision.
d) Now suppose that the paint fleck bounced off the astronaut and reverses direction with speed $8.0 \times 10^{3} \mathrm{~m} / \mathrm{s}$. Determine the speed of the astronaut after the collision.

## 138 Sledder catching a ball

A person sits at rest on a sled on a sheet of ice; their combined mass is 95.0 kg . A 5.0 kg bowling ball is launched horizontally rightwards toward the person with speed $20 \mathrm{~m} / \mathrm{s}$. The person catches the ball and the person, sled and ball slide together at the same speed.
a) Determine their velocity after the ball has been caught.
b) Is kinetic energy conserved in this process?

Suppose that the person and sled had been moving left with speed $2.0 \mathrm{~m} / \mathrm{s}$ prior to catching the ball.
c) Determine their velocity after the ball has been caught.

## 139 Dropping a rock on a planet

An astronaut stands on a planet, holding a rock at rest above the surface of the planet. The planet, astronaut and rock are initially at rest. The astronaut releases the rock and it falls toward the planet.
a) While the rock falls is the total momentum of the planet/astronaut/rock system zero or not? Explain your answer.
b) While the rock falls does the planet remain at rest or not? Explain your answer.

## 140 Separating carts

Two carts are at rest on a frictionless horizontal track. The mass of cart A is exactly four times the mass of cart B. A spring between the carts is released, the carts separate and cart A moves left with speed $v$.
a) Which of the following is true for cart B after the collision? Explain your answer
i) B moves left with speed $v$.
ii) B moves left with speed $2 v$.
iii) B moves left with speed $4 v$.
iv) B moves right with speed $v$.
v) B moves right with speed $2 v$.
vi) B moves right with speed $4 v$.
b) Which cart gains the most kinetic energy? Explain your answer.

## 141 Ballistic pendulum

A ballistic pendulum consists of a 1.5 kg wooden block suspended at rest from a cord. A 0.0080 kg bullet is fired horizontally into the block. It embeds itself causing the block to swing upward, reaching a point 0.20 m above its lowest point. The idea is to use the maximum height reached by the pendulum to determine the speed of the bullet immediately before hitting the block.The process is illustrated at three instants. Let instant 1 be the moment just before the bullet hits,
 instant 2 , the moment just after it embeds and instant 3 , the moment at which it reaches its highest point.
a) Is momentum conserved from instant 1 to instant 2? Is mechanical energy, $E=K+$ $U_{\text {grav }}$, conserved from instant 1 to instant 2?
b) Is momentum conserved from instant 2 to instant 3? Is mechanical energy, $E=K+$ $U_{\text {grav }}$, conserved from instant 2 to instant 3?
c) Consider the period from instant 2 to instant 3 and use this to determine the speed of the bullet and block just after it embeds.
d) Consider the period from instant 1 to instant 2 and use this to determine the speed of the bullet and block just before it embeds.

## 142 Ballistic pendulum

A 0.010 kg bullet is fired horizontally into a 2.0 kg block that is suspended at rest from a string just above the ground. The bullet collides with the block and sticks. The two then swing in a circle and the block reaches a height of 0.30 m above the ground.
a) Determine the speed of the bullet and block just after the bullet has stuck to the block.
b) Determine the speed of the bullet just before the bullet hits the block.

## 143 Half pipe

A 60 kg person rides a 5.0 kg skateboard. Both are at rest at the top of a half-pipe of radius 10 m . Ignore any friction and the rotation of the wheels.
a) Determine the speed of the skateboarder at the bottom of the half pipe.
b) Suppose that there was another skateboarder, of
 mass 80 kg on a 5.0 kg skateboard at rest at the bottom of the pipe. The two skateboarders collide, hold each other and move together. Determine their speed moments after they collide
c) Determine how high up the pipe the two combined skateboarders move.

## Rotational Motion

## 144 Rotating bar bell

A rigid barbell rotates about the axle at $O$. The distance from the end labeled B to $O$ is three times the distance
 from the end labeled A to $O$.
a) Which of the following is true? Explain your answer.
i) The linear speed of B is nine times that of A .
ii) The linear speed of $B$ is three times that of $A$.
iii) The linear speed of $B$ is the same as that of $A$.
iv) The linear speed of $B$ is one third that of $A$.
v) The linear speed of $B$ is one ninth that of $A$.
b) Explain as precisely as possible how the centripetal acceleration of A is related (e.g. same, four times, ...) to that of B.

## 145 Bugs on a rotating disk

The PhET animation
http://phet.colorado.edu/en/simulation/rotation
illustrates bugs on a rotating disk. Use the link to run the animation. Once the animation has opened, use the Intro tab (this is the default tab). The two bugs can be moved to various parts of the disk and the disk can be made to rotate. Place one bug a quarter of the way from the center to the edge. Place the other half way from the center to the edge.
a) Predict how many times larger (or smaller) the velocity of the bug that is closer to the center is than that of the bug further from the center. Explain your prediction.
b) Predict how many times larger (or smaller) the centripetal acceleration of the bug that is closer to the center is than that of the bug further from the center. Explain your prediction.
c) Run the animation by entering various angular velocities. It should display the velocity and acceleration vectors. Do your results for the previous parts agree with the observed acceleration vectors? Do your predictions from the previous parts depend on the angular velocity?

## 146 Hard drive rotation

A hard drive in a computer contains a disk that rotates at a rate of 7200 rpm . The diameter of the disk is 6.25 cm .
a) Determine the speed of a point at the edge of the disk while it is rotating at 7200 rpm .
b) Assume that the disk is initially at rest and accelerates at a constant rate reaching 7200 rpm in 4.1 ms . Determine the angular acceleration of the disk.

## 147 Rotating bicycle wheels

A bicycle travels with speed $6.5 \mathrm{~m} / \mathrm{s}$ (about 15 mph ). The diameter of the bicycle wheel is 0.700 m . Determine the angular velocity of the wheel.

## 148 Torques on a beam

A beam can pivot about its left end. Various forces act on the beam as illustrated. An axis is provided for scale. The magnitudes of the forces are related via $F_{2}=3 F_{1}, F_{3}=2 F_{1}$, and $F_{4}=F_{1}$. Rank the torques produced by the forces about the left end of the beam in order of increasing torque. Explain your answer.

## 149 Beam in equilibrium

A 4.0 m long, 80 kg beam is anchored to a wall and held at rest horizontally by a rope at the illustrated angle. A 10 kg box rests on the beam at the illustrated point. The aim of this exercise is to determine the tension in the rope. This would enable one to decide on the breaking strength of the rope.


Wall

a) State the conditions for equilibrium.
b) Draw all the force vectors on the beam.
c) Identify a pivot point (there are many correct possibilities - one is much more useful than the others) and determine expressions for the torque exerted by each force about the pivot.
d) Substitute the individual torques into one of the conditions for equilibrium and obtain an expression for the tension in the rope.

## 150 Beam in equilibrium

A 10 kg beam with length 2.0 m can pivot about its left end. A rope is attached as illustrated. Determine the tension so that the beam stays at rest horizontally.
a) State the conditions for equilibrium.
b) Draw all the force vectors on the beam.
c) Identify a pivot point (there are many correct possibilities - one is much more useful than the others) and determine expressions for the torque ex-
 erted by each force about the pivot.
d) Substitute the individual torques into one of the conditions for equilibrium and determine the tension in the rope.

## 151 Beam in equilibrium

A beam with mass $M$ and length $L$ is anchored to a wall and held at rest horizontally by a rope as illustrated. A ball with mass $m$ is suspended from the beam at the illustrated point. The aim of this exercise is to determine the tension in the rope. This would enable one to decide on the breaking strength of the rope.
a) State the conditions for equilibrium.
b) Draw all the force vectors on the beam.
c) Identify a pivot point (there are many correct possibilities - one is much more useful than the others) and determine expressions for the torque exerted by each force about the pivot.
d) Substitute the individual torques into one of the conditions for equilibrium and obtain an expression for the tension in the rope.
e) What must be the minimum required breaking tension for the rope if it is to support a 40 kg beam with length 3.0 m while there is a 8.0 kg ball suspended as illustrated?

## 152 Balance beam

A 60 kg person stands on a 10 kg horizontal beam, with length 3.0 m , at the point illustrated by the dot. The beam is supported by stands at either end and is in equilibrium. Each stand exerts an upward force.
a) Determine the force provided by the support on the right. Hint: Consider torques about the left support and don't forget the mass of the beam.
b) Determine the force provided by the support on the left. Hint: Consider torques about the right support and don't forget the mass of the beam.

## 153 Rotating disk

Various forces act on a 0.50 kg disk with radius 0.15 m . The disk can rotate about an axle through its center and perpendicular to the disk. The 30 N force acts at a point 0.10 m from the edge of the disk. The 10 N force acts at a point 0.08 m from the axle.
a) Determine the torque about the axle produced by each force on the disk.
b) Determine net torque on the disk.
c) Determine the angular acceleration of the disk.


## 154 Moments of Inertia

Two circular disks are made of the same material. Disk A has a radius three times that of disk B ; their thicknesses are the same. Let $I_{A}$ be the moment of inertia of A and $I_{B}$ be the moment of inertia of B (each about an axis perpendicular to the page). Which of the following is true? Explain your answer.
i) $I_{A}=I_{B}$
ii) $I_{A}=\frac{9}{2} I_{B}$
iii) $I_{A}=9 I_{B}$
iv) $I_{A}=\frac{81}{2} I_{B}$
v) $I_{A}=81 I_{B}$

## 155 Moment of inertia about different axes

A flat equilateral triangular plate can rotate about one of two axes. Point A is at the center and point B is at one of the corners. Let $I_{A}$ be the moment of inertia about A and $I_{B}$ be the moment of inertia about B (each about an axis perpendicular to the page). Which of the following is true? Explain your answer.
i) $I_{A}=I_{B}$
ii) $I_{A}>I_{B}$
iii) $I_{A}<I_{B}$

## 156 Moment of inertia for point masses

Three point masses are connected by massless rods. Determine the moment of inertia about an axis perpendicular to the page and that passes through:
a) the 150 g mass,
b) the 100 g mass, and
c) the 200 g mass.


## 157 Rotating disk

A 2.0 kg turntable (disk) has radius 0.10 m and can rotate horizontally about a frictionless axle through its center. A 1.2 kg blob of putty is stuck to a point on the disk three quarters of the distance from the center to the edge. A jet attached halfway from the center to the edge of the disk exerts a 4.0 N tangential force as illustrated. The aim of this exercise is to determine the angular acceleration of the disk via the following steps.

a) Write the rotational version of Newton's second law.
b) Determine the moment of inertia of the disk plus putty.
c) Determine the net torque acting on the disk.
d) Determine the angular acceleration of the disk.
e) Suppose that a brake pad presses on the rim of the disk, producing a frictional force with magnitude 1.5 N while the jet is operating. If the wheel is rotating counterclockwise, determine the angular acceleration of the disk in this situation.

## 158 Rotating rod

A rod with length $L$ lies on a frictionless horizontal surface and can rotate about an axle at its left end. In separate instances one force is applied to the rod as illustrated. The angles between forces B and C and the rod are the same. Rank the magnitude of the angular accelerations in order from largest to smallest. Explain your answer.


## 159 Rotational kinetic energy

Various spheres have the same mass and rotate about a diametrical axis with different angular velocities as listed in the table.

| Object | type | radius | $\omega$ |
| :---: | :---: | :---: | :---: |
| A | Solid sphere | 1.0 m | $8 \mathrm{rad} / \mathrm{s}$ |
| B | Solid sphere | 2.0 m | $4 \mathrm{rad} / \mathrm{s}$ |
| C | Hollow sphere | 2.0 m | $4 \mathrm{rad} / \mathrm{s}$ |
| D | Hollow sphere | 3.0 m | $2 \mathrm{rad} / \mathrm{s}$ |

Rank these in order of increasing rotational kinetic energy. Explain your answer.

## 160 Mass suspended from a rotating wheel

A 10 kg solid wheel with radius 0.40 m can rotate about a frictionless axle through its center. A 5.0 kg block is suspended from a string which is wrapped around the wheel. The system is held at rest with the block 2.0 m above the ground. The block is then released, causing the wheel to rotate (it's axle does not drop).
a) Determine the total energy of the system at the moment that the block is released. What type of energy is this?

b) What type of energy does the system have at the instant just before the block hits the ground? Determine an expression for the total energy of the system at this instant; the expression should include the speed of the block, $v$, and the angular velocity of the wheel $\omega$.
c) If the string does not slip along the wheel the speed of the block $v$ satisfies $v=\omega r$ where $r$ is the radius of the wheel. Substitute this into the result of the previous part to show that the total energy just before the block hits the ground is

$$
E=\frac{1}{2} m_{\mathrm{block}} r^{2} \omega^{2}+\frac{1}{2} I_{\text {wheel }} \omega^{2} .
$$

Use this to determine the angular velocity of the wheel just before the block hits the ground.
d) Determine the speed of the block just before it hits the ground.

## 161 Rolling and sliding cylinders

Two cylinders with the same mass and radius can roll along a ramp. Both cylinders roll without slipping along the horizontal section at the base of the ramp. Along
 the sloped portion of the ramp, cylinder A encounters no friction and slips, but cylinder B rolls without slipping.
a) Both cylinders are released from rest at the top of the ramp. Is the speed of A at the bottom of the ramp the same, larger than or smaller than that of B at the bottom of the ramp? Explain your answer.
b) The cylinders are launched along the horizontal surface so that they arrive at the bottom of the ramp with the same velocity and the same angular velocity. Is the maximum height attained by A the same, larger than or smaller than that of B? Explain your answer.

## 162 Rolling ball

A 0.250 kg hollow ball with radius 0.10 m is held at rest at the top of a ramp that is 3.0 m long. The ramp is inclined at an angle of $15^{\circ}$ above the horizontal. The ball is released and rolls without slipping.
a) Determine the angular velocity of the ball at the base of the ramp.
b) Determine the speed of the ball at the base of the ramp.
c) Does the speed of the ball at the base of the ramp depend on the mass of the ball? Explain your answer.
d) Does the speed of the ball at the base of the ramp depend on the radius of the ball? Explain your answer.

## 163 Rotational collision

A 0.150 kg solid disk with radius 0.050 m rotates at 180 revolutions per minute. A 0.400 kg ring with radius 0.030 m is held at rest and then gently dropped onto the disk so that its center coincides with the center of the disk. It sticks. Determine the angular velocity of the combination after the ring sticks to the disk.

## 164 Turntable and blocks

A circular 20 kg turntable with radius 0.50 m rotates at 240 rpm in a horizontal plane about an axle through its center. The mass of the turntable is uniformly distributed. Two small blocks, each with mass 5.0 kg are gently lowered onto the turntable and eventually stick at locations that are $3 / 4$ of the distance from the axle to the rim.
a) Determine the angular velocity of the turntable after the blocks have settled on the turntable.
b) Determine the change in kinetic energy of the system from the moment before the blocks hit the turntable until they settle.


## 165 Merry-go-round

A merry-go-round is a large circular turntable which rotates horizontally about an axle through its center. Children can stand and move about on the merry-go-round. A particular merry-go-round has radius 2.0 m and moment of inertia $600 \mathrm{kgm}^{2}$. A 50 kg child stands at the edge of the merry-go-round and her parent pushes it so that it rotates at a constant rate of 40 rpm . The child then walks toward the center of the merry-go-round. Approximating the child as a point particle, determine the angular velocity of the merry-go-round when she is 0.30 m from the axle.

## 166 Elliptical orbit of a satellite

A satellite moves around a planet in an elliptical orbit. At all times the force exerted by the planet on the satellite points directly toward the planet. The satellite's motion can be analyzed by thinking of it as a point particle in a rotational orbit about the point where the planet is located.

a) Is the net torque (about the planet's location) on the satellite zero or not at all times? Explain your answer.
b) Is the angular momentum (about the planet's location) of the satellite constant at all times? Explain your answer.
c) Determine an expression for the angular momentum of the satellite in terms of its mass, speed and distance from the planet.
d) Using angular momentum, describe whether the satellite is moving faster or slower at location A in comparison to location B .

## Thermal Physics

## 167 Temperature scales

a) The highest recorded temperature on Earth's surface was $134.1^{\circ} \mathrm{F}$, which was recorded in Death Valley National Park in California. Determine this temperature in degrees Celsius and Kelvin.
b) The boiling point of liquid Helium-4 is 4.2 K . Determine this temperature in degrees Celsius and degrees Fahrenheit.

## 168 Air pressure on a hand

A 65 kg person is in a location where the air pressure is 100 kPa . The person's hand can be regarded as a rectangle with length 18 cm and width 8.0 cm . The had is held out so that it is horizontal.
a) Determine the force exerted by the air above the hand on the hand. How does this compare to the gravitational force exerted by Earth on the person?
b) Explain why despite the force exerted by the air above the hand, why it is relatively easy for the person to hold their hand up?

## 169 Box of gas

A cubic box has sides 0.50 m long. The box is in a room whose temperature is $23^{\circ} \mathrm{C}$.
a) Suppose that the box is at sea level where the pressure is 101 kPa . Determine the number of moles of gas inside the box. Determine the number of gas particles in the box.
b) Suppose that the box is at an altitude comparable to Grand Junction where the pressure is 87 kPa . Determine the number of moles of gas inside the box.

## 170 Venus' atmosphere

Venus has a surface temperature of about 737 K and a pressure about 91 times that of Earth's atmosphere. The atmosphere is made up mostly of Carbon Dioxide $\left(\mathrm{CO}_{2}\right)$.
a) Determine the number of moles of gas in Venus' atmosphere per meter cubed at the surface of Venus.
b) Determine the density (mass per meter cubed) of Venus' atmosphere. How does this compare to the density of Earth's atmosphere (about $1.2 \mathrm{~kg} / \mathrm{m}^{3}$ )?

## 171 Ideal gas constant volume process

An ideal gas is initially at temperature $40^{\circ} \mathrm{C}$. Heat is supplied to the gas while its volume is held constant. At the end of this process the pressure is double what it had been initially. Which of the following is true about the temperature at the end of the process, $T_{f}$ ?
i) $T_{f}=40^{\circ} \mathrm{C}$
ii) $T_{f}=80^{\circ} \mathrm{C}$
iii) $T_{f}$ is somewhere between $40^{\circ} \mathrm{C}$ and $80^{\circ} \mathrm{C}$.
iv) $T_{f}$ is more than $80^{\circ} \mathrm{C}$

Explain your answer.

## 172 Ideal gas: isothermal process

An isothermal process is one in which the temperature of a gas stays constant while its pressure and volume change. An ideal gas is initially at pressure $P_{i}$ and has volume $V_{i}$. The gas is allowed to expand in an isothermal process so that its volume eventually reaches $4 V_{i}$. Which of the following gives its pressure at this point?
i) $\frac{P_{i}}{4}$
ii) $\frac{P_{i}}{2}$
iii) $P_{i}$
iv) $2 P_{i}$
v) $4 P_{i}$

Explain your answer.

## 173 Pressures of distinct gases

Several containers all have the same volume but each contain distinct gases at the same temperature. The information about these gases is given below:

| Container | Gas | Mass per mole | Number of moles |
| :---: | :---: | :---: | :---: |
| A | Oxygen | 32 g | 2 |
| B | Hydrogen | 2 g | 16 |
| C | Carbon Monoxide | 28 g | 2 |
| D | Carbon Monoxide | 28 g | 1 |

Rank these gases in order of increasing pressure, indicating equality whenever applicable. Explain your answer.

## 174 Energy and temperature

Neon is a monoatomic gas. A bulb holds 0.025 mol of Neon gas and this is initially at temperature $40^{\circ} \mathrm{C}$.
a) The temperature is increased to $50^{\circ} \mathrm{C}$. Determine the amount by which the energy increased as a result of the temperature increase.
b) Suppose that, with the gas initially at temperature $40^{\circ} \mathrm{C}$, the gas undergoes a process which increases its energy by $50 \%$. Determine the temperature of the gas at the end of this process.

## 175 Energy of a balloon of gas

A balloon is filled with Helium, a monoatomic gas, and at sea level atmospheric pressure and temperature $20^{\circ}$ it forms a sphere with radius 0.50 m .
a) Determine the thermal (internal) energy of the gas in the balloon.
b) To get an idea of the energy in this balloon, suppose that a person is initially at rest on a skateboard. The mass of the person and skateboard is 70 kg . The imagine that all of the energy is the balloon was somehow used to get the person to move and the kinetic energy of the person and skateboard was equal to the thermal energy of the gas. Determine the resulting speed of the person and skateboard in miles per hour.

## 176 Gold and water

A 10.0 kg piece of gold is initially at $100^{\circ} \mathrm{C}$. It is immersed in 1.00 kg of water which is initially at $20.0^{\circ} \mathrm{C}$. The aim of this exercise is to determine temperature, $T_{f}$ of the gold and water after they have reached equilibrium. The heat capacity of gold is $129 \mathrm{~J} / \mathrm{kg} \mathrm{K}$ and that of water is $4190 \mathrm{~J} / \mathrm{kg} \mathrm{K}$.
a) Determine an expression for the heat lost by the gold in terms of $T_{f}$ and the data provided.
b) Determine an expression for the heat gained by the water in terms of $T_{f}$ and the data provided.
c) Combine your expressions to determine the equilibrium temperature $T_{f}$.

## 177 Heating water

A (typical) cup holds 0.250 kg of water. Suppose that the water is initially at temperature $20^{\circ} \mathrm{C}$. The water is then heated by a kettle that provides a power of 750 W .
a) Assuming that the heat transfer from the kettle to the water is perfectly efficient, determine the increase in the water temperature per second of heating.
b) Determine the amount of time needed to heat the water to a temperature of $80^{\circ} \mathrm{C}$.

## 178 Heating water

Two styrofoam cups, one red and the other blue, contain the same amount of water. The red cup is initially at $70^{\circ} \mathrm{C}$ and cools to $40^{\circ} \mathrm{C}$, while the blue cup is initially at $90^{\circ} \mathrm{C}$ and cools to $60^{\circ} \mathrm{C}$. The effect of the cups on the cooling is negligible. Which of the following is true?
i) The water in the red cup loses the same heat as that in the blue cup.
ii) The water in the red cup loses more heat than that in the blue cup.
iii) The water in the red cup loses less heat than that in the blue cup.

## Explain your answer.

## 179 Cooling with a fluid

Suppose that you would like to cool a piece of copper initially at $70^{\circ} \mathrm{C}$. You can do this by immersing the copper in equal masses of water or ethanol (ethyl alcohol), both initially at $20^{\circ} \mathrm{C}$. Which of these fluids will result in the larger reduction in copper temperature? Explain your answer.

## 180 Forces at the ocean floor

The deepest point beneath sea level is 10994 m below sea level. A cubic box with sides 2.0 m lone is lowered to this point. Determine the force exerted by the water on one side of the box. How many times larger is this than the force exerted by Earth's atmosphere on this side when the box is at sea level?

## 181 Buoyant forces

A container is filled with water and there objects are immersed in the container as illustrated (the block sizes are not to scale). The mass of A is 80 g , the mass of $B$ is 60 g and the mass of C is 80 g .
a) Suppose that the volumes of the blocks are all the same. Rank the buoyant forces on each block in order of increasing magnitude.
b) Suppose that the densities of the blocks are all
 the same. Rank the buoyant forces on each block in order of increasing magnitude.

## 182 Raising an object through water

A 20 kg treasure chest is raised by a rope at constant speed from the bottom of a lake (density $\left.1000 \mathrm{~kg} / \mathrm{m}^{3}\right)$. The volume of the chest is $12 \times 10^{-3} \mathrm{~m}^{3}$.
a) Determine the tension in the rope while the chest is being raised and is still completely submerged. Note that this involves Newton's second law and the solution should include the usual ingredients used in applying Newton's second law, i.e. a free body diagram, etc,...
b) The chest eventually begins to emerge from the water and is held at rest with half of its volume still submerged. Determine the tension in the rope at this point.

## Oscillations

## 183 Frequency of oscillation

A mechanical device contains an part that oscillates with frequency 250 Hz .
a) Determine the number of cycles that the part completes in 1.0 minute.
b) Determine the number of cycles that the part completes in 1.0 hour.
c) Determine the period of oscillation.

## 184 Oscillating spring and mass frequency

A block is attached to a spring and allowed to slide back and forth across a frictionless horizontal surface. The spring has constant $5.00 \mathrm{~N} / \mathrm{m}$. Determine the mass that must be attached to the spring so that the frequency of oscillation is 20 Hz .

## 185 Oscillating spring and mass

A 0.125 kg block is suspended from a spring. The block oscillates, taking 3.0 s to complete 10 cycles.
a) Determine the spring constant of the spring.
b) The block is replaced by another with mass 0.500 kg . Determine the amount of time that this takes to complete 10 cycles of oscillation.

## 186 Children on swings

Two children swing on swings whose seats are attached by ropes to a tree branch above the children. The lengths of the ropes are the same. One child, Jordi, has mass 40 kg and the other, Montserrat, has mass 30 kg . Which of the following is true? Consider the swing to be a pendulum oscillating with small amplitude.
i) The time for Jordi to swing back and forth once is the same as that for Montserrat.
ii) The time for Jordi to swing back and forth once is larger than that for Montserrat.
iii) The time for Jordi to swing back and forth once is smaller than that for Montserrat.

## Waves

## 187 Wavelength and frequency

Various waves on strings are as illustrated. The wave speeds are provided for each case. The units of the axes are meters.
a) Rank the waves in order of increasing wavelength. Indicate equality whenever it occurs. Explain your answer.
b) Rank the waves in order of increasing frequency. Indicate equality whenever it occurs. Explain your answer.


## 188 Waves on a string

A snapshot of a wave on a string is illustrated. The units of the axes are meters.
a) Determine the wavelength of the wave.
b) The wave is observed as time passes and it is found that 20 crests pass the 4 m mark in 5.0 s . Determine the wave speed.


## 189 Waves on a string with an oscillating end

The PhET animation "Waves on a String" allows you to visualize transverse waves on a string. Find the animation at

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https://phet.colorado.edu/en/simulation/wave-on-a-string
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and open it. Adjust the settings as follows:

1. Check the button "No end" at the upper right.
2. Check the button "Oscillate" at the upper left.
3. In the control panel at the bottom, adjust "Damping" to none.
4. In the control panel at the bottom, adjust "Tension" to low.
5. In the control panel at the bottom, check the "Rulers" and "Timer" buttons.
a) Set the frequency to 0.30 Hz . Using the rulers and timer, determine the speed of the wave. Using the ruler, determine the wavelength of the wave. Determine the frequency of the wave by counting crests that pass a given point.
b) Set the frequency to 0.60 Hz . Repeat part a).
c) Is the speed the same in both cases?
d) Check that $v=\lambda f$ in both cases.
e) For one of the frequencies, vary the amplitude and show that it does not affect the wave speed or wavelength.

## 190 Sinusoidal waves

A snapshot of a segment of a wave on a string at a particular instant is illustrated. The distances are measured in meters.

a) The portion of the wave at A takes 0.0050 s to complete one cycle. Determine the frequency of the wave.
b) Determine the wavelength and speed of the wave.
c) Determine how long it will take the crest labeled B to reach the point C .
d) Using the graph above, sketch the wave at an instant 0.0025 s after that illustrated above.

## 191 Waves on a stretched string

A wire string has length 1.25 m and mass 0.0025 kg . It is stretched horizontally between supports that are 1.00 m apart, with the remaining portion of wire dangling vertically beyond one of these supports. A 0.400 kg mass is suspended from this portion of wire. Possible wavelengths of waves on this string are $2.00 \mathrm{~m}, 1.00 \mathrm{~m}, 0.667 \mathrm{~m}$. Determine the frequency of each of these waves.

