Phys 396 Spring 2018

Lecture 3

Tues: HW by 5pm

Thurs: Seminar

Read... B 2.3 pg 57 - 62

R 2.2 → 2.3

Quantum Physics

Quantum information processing uses quantum physics systems to represent and process information. The means by which information can be represented and processed refers to the fundamental rules of quantum theory. We shall develop these rules in the context of so-called spin-½ systems. To do so we shall present:

1) connections between spin angular momentum and magnetic dipole moment

2) the Stern-Gerlach experiment for spin-½ systems

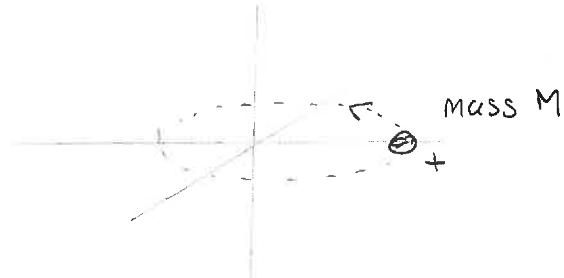
3) cascades of Stern-Gerlach experiments

Spin-angular momentum and magnetic dipole moment

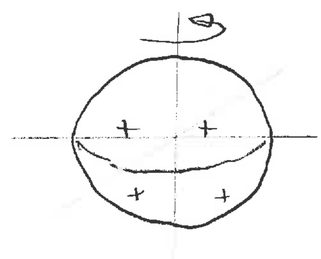
Consider classical rotating charge distributions. These will produce both an angular moment and a magnetic dipole moment.

Possible examples include:

1) orbiting point charge



2) rotating shell of charge



In each case we expect that the (spin) angular momentum is related to the magnetic dipole moment. For example consider a point charge orbiting with constant speed v at radius r

Angular momentum	Magnetic Dipole Moment
$\vec{L} = \vec{r} \times \vec{p}$	$\vec{\mu} = \frac{1}{2}\int \vec{r}\,' \times \vec{J}(\vec{r}\,')\, d\tau'$
$\vec{L} = rMv\ \hat{z}$	$\Rightarrow \vec{\mu} = I\vec{A}$
	$\Rightarrow \vec{\mu} = \frac{Q}{T}\pi r^2\ \hat{z}$
	$= \frac{Q}{(2\pi r)/v}\pi r^2\ \hat{z}$
	$\Rightarrow \vec{\mu} = \frac{Qvr}{2}\ \hat{z}$

Thus we see that

$$\boxed{\vec{\mu} = \frac{Q}{2M}\vec{L}}$$

In general, we can extend this to situations where a charge distribution is spinning. We replace orbital angular momentum by spin angular momentum $\vec{S}$. Additionally there is a "g-factor" that depends on how the mass and charge distributions are related. This gives:

$$\boxed{\vec{\mu} = \frac{gQ}{2M} \vec{S}}$$

The most important consequence of this for us that measuring any component of the magnetic dipole moment allows us to compute the associated component of spin angular momentum. Roughly

$$\boxed{\text{Measuring spin} \sim \text{measuring magnetic dipole moment}}$$

Since we can determine spin by measuring magnetic dipole moment, we need a scheme for doing the latter. We can do this by placing a particle in an appropriately designed magnetic field. Then with the potential energy

$$U = -\vec{\mu} \cdot \vec{B}$$

and force

$$\vec{F} = -\vec{\nabla} U$$

Consider a field along the z-direction

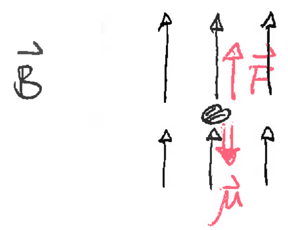
we get

$$F_x = -\mu_z \frac{\partial B_z}{\partial x}$$

$$F_y = -\mu_z \frac{\partial B_z}{\partial y}$$

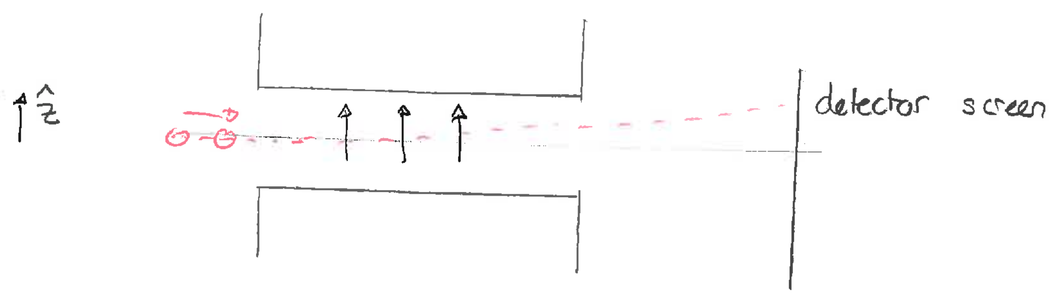
$$F_z = -\mu_z \frac{\partial B_z}{\partial z}$$

To measure μ_z we can place the particle in a magnetic field with a gradient in the z-direction:



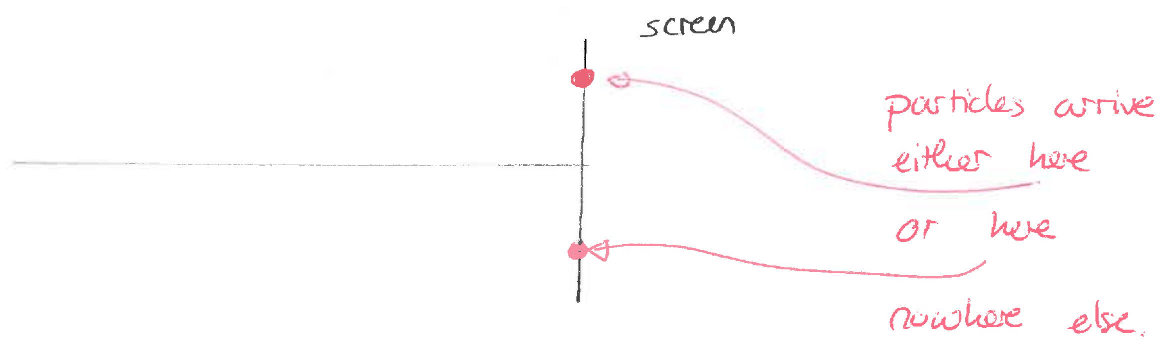
Stern-Gerlach experiment.

The Stern-Gerlach experiment operates by firing particles into a magnetic field with a gradient along one direction.



The deflection on the detector turns out to be proportional to the component of the spin along the magnetic field direction.

Suppose that the field is along the $+\hat{z}$ direction. What the experiment shows is that there are only two possible deflections.
↳ for certain particles



When converted to appropriate units we find that this SG experiment yields one of two possible outcomes:

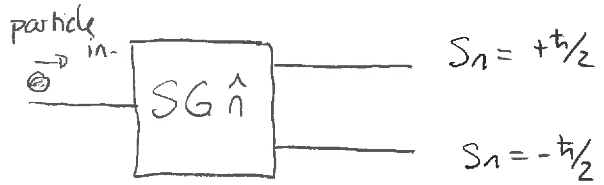
$$S_z = +\hbar/2 \quad \text{OR} \quad S_z = -\hbar/2$$

We could rotate the entire apparatus so that the field points along the y direction. We would find that either $S_y = +\hbar/2$ or $S_y = -\hbar/2$.

A particle for which the SG experiment yields such outcomes is called a spin-½ particle. In general

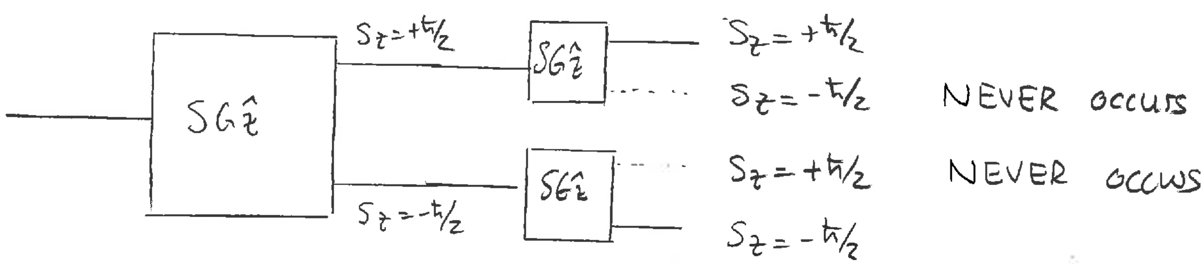
> If a SG experiment is configured to measure the component of spin along $+\hat{n}$ then the outcome will be one of
>
> $$S_n = +\hbar/2 \quad \text{OR} \quad S_n = -\hbar/2$$
>
> where S_n is the component of spin along $\hat{n}$.

We can denote this schematically as



<u>Repeated SG measurements</u>

Consider spin-½ particles that are subjected to a succession of SG measurements. First suppose that the SG measurements are of the same type, for example both along $+\hat{z}$. What occurs? We find



Thus if the first SG$\hat{z}$ yields $S_z = +\hbar/2$ then the second yields $S_z = +\hbar/2$ with certainty. Similarly with $S_z = -\hbar/2$ for the pair. Thus we know something about the state of the system emerging from the first SG$\hat{z}$ (if we know the measurement outcome). We use the notation for the state of a spin-½ particle:

$$|+\hat{z}\rangle \iff \text{SG}\hat{z} \text{ yields } +\hbar/2 \text{ with certainty}$$
$$|-\hat{z}\rangle \iff \text{SG}\hat{z} \quad '' \quad -\hbar/2 \quad '' \quad ''$$

The symbol $|\text{label}\rangle$ is called a "ket"

More generally we have

$\|+\hat{n}\rangle$	$\Leftrightarrow$	$SG\hat{n}$	yields	$S_n = +\hbar/2$	with certainty
$\|-\hat{n}\rangle$	$\Leftrightarrow$	$SG\hat{n}$	"	$S_n = -\hbar/2$	" "

This is the operational meaning of such quantum states. Note that these have a generic form:

$$\underbrace{|\psi\rangle}_{\text{label}}$$ means that there is some measurement so that if the particle were subject to that measurement then it would yield one particular result with certainty.

The label describes the measurement and associated outcome.

Phys 396
Fall 2018

Quantum Information: Class 3

28 August 2018

1 Single Stern-Gerlach experiments

A stream of particles are all prepared in an identical fashion. These are incident upon a SG $\hat{\boldsymbol{y}}$ apparatus. About 60% of these yield $S_y = +\hbar/2$ and about 40% of these yield $S_y = -\hbar/2$.

a) Based on this could one say that the state of every particle after it has been prepared and before it enters the SG apparatus is $|+\hat{\mathbf{y}}\rangle$?

b) Could one say that the state of every particle after it has been prepared and before it enters the SG apparatus is $|-\hat{\mathbf{y}}\rangle$?

c) Consider any particle that yields $S_y = +\hbar/2$. After this emerges from the apparatus, can one say what it's state is? Explain your answer.

d) Consider any particle that yields $S_y = -\hbar/2$. After this emerges from the apparatus, can one say what it's state is? Explain your answer.

e) Describe how one might take a particle in some unknown state and use a SG apparatus to prepare the particle in state $|-\hat{\mathbf{x}}\rangle$. When will the procedure definitely fail?

Answer:

a) No. If the state were $|+\hat{y}\rangle$ every measurement would give $+\hbar/2$. It does not

b) No. Then every measurement would give $-\hbar/2$. It does not

c) Yes. Another SG $\hat{y}$ will give $+\hbar/2$ with certainty. Thus the state is $|+\hat{y}\rangle$

d) Similar to c) but the state is $|-\hat{y}\rangle$

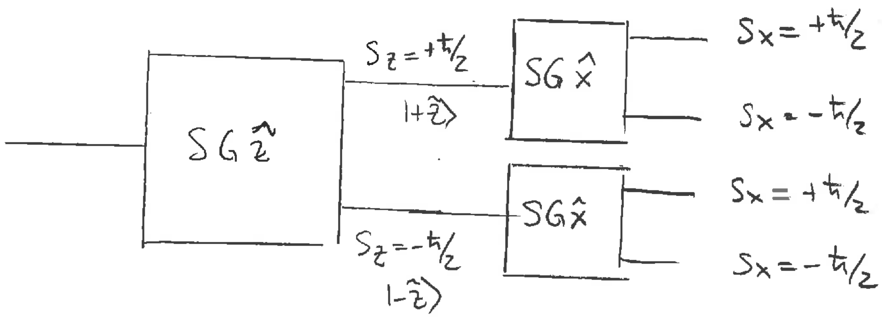
e) Subject it to SG$\hat{x}$. If we get $S_x = -\hbar/2$ then state is $|-\hat{x}\rangle$. This will definitely fail if the state entering is $|+\hat{x}\rangle$

We thus see that we can determine the state after measurement:

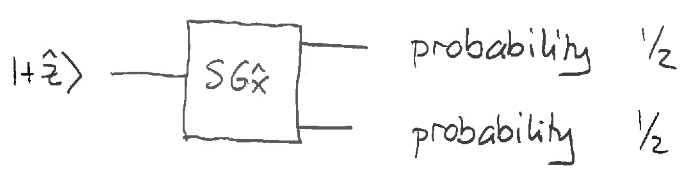
emerges here $\Rightarrow$ state is now $|+\hat{n}\rangle$

Repeated measurements of different types.

Consider an SG$\hat{z}$ followed by SG$\hat{x}$. We find that there are four possible pairs of outcomes and none is excluded

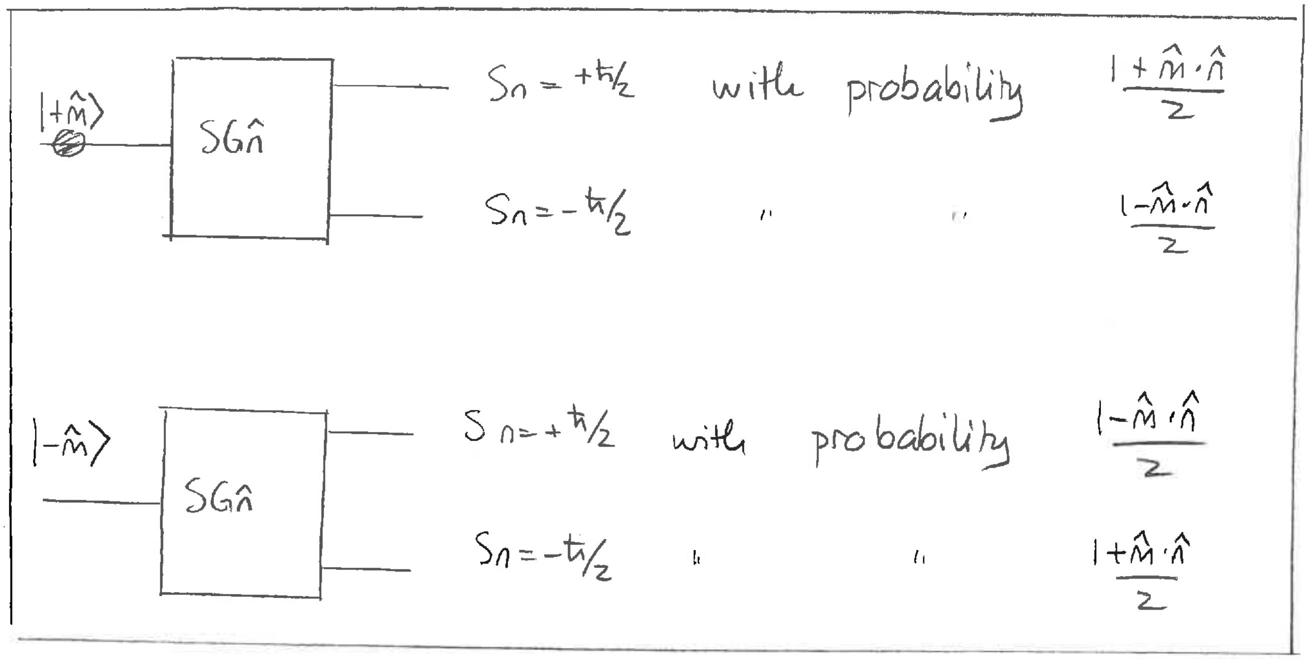


In any given run, the particle will only emerge in one final output. But even if it is prepared (prior to SG$\hat{z}$) in the same way it will not always emerge in the same location. We do know that if it emerges with $S_z = +\hbar/2$ then prior to SG$\hat{x}$ its state was $|+\hat{z}\rangle$. What we can get from experiments is:



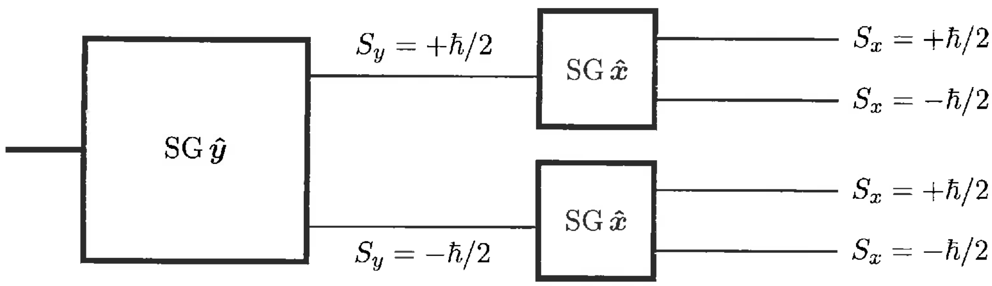
We can do such experiments for all input states and all SG choices

We find, for any unit vector directions



2 Repeated Stern-Gerlach measurements

Two successive SG measurements are oriented as illustrated.

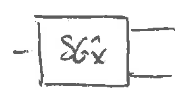


a) Suppose that the particle emerges from the first with $S_y = +\hbar/2$. Determine the probabilities of the two outcomes of the SG $\hat{x}$ measurement.

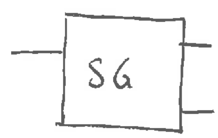
b) Suppose that the particle emerges from the first with $S_y = -\hbar/2$. Determine the probabilities of the two outcomes of the SG $\hat{x}$ measurement.

c) Suppose that the state of the particle prior to the first is $|+\hat{\mathbf{n}}\rangle$ where $\hat{\mathbf{n}} = (\hat{\mathbf{x}} + \hat{\mathbf{y}})/\sqrt{2}$. Determine the probabilities with which it emerges in all four outputs.

Answer: a) Then the state after SG $\hat{y}$ is $|+\hat{y}\rangle$. Thus

$|+\hat{y}\rangle$ 

$$\text{prob} = (1 + \hat{x}\cdot\hat{y})/2 = 1/2$$
$$\text{prob} = (1 - \hat{x}\cdot\hat{y})/2 = 1/2$$

b) $|-\hat{y}\rangle$ 

$$\text{prob} = (1 - \hat{x}\cdot\hat{y})/2 = 1/2$$
$$\text{prob} = (1 + \hat{x}\cdot\hat{y})/2 = 1/2$$

c) After the first

outcome:	state after	Prob
$S_y = +\hbar/2$	$\lvert+\hat{y}\rangle$	$(1 + 1/\sqrt{2})/2$
$S_y = -\hbar/2$	$\lvert-\hat{y}\rangle$	$(1 - 1/\sqrt{2})/2$

Now suppose that the first yields $S_y = +\hbar/2$. Then the state is $|+\hat{y}\rangle$ and this enters $SG\hat{x}$. Given this Prob $(S_x = +\hbar/2) = 1/2$ and prob $(S_x = -\hbar/2) = 1/2$. So we get

First SG	Second SG	Joint prob.
$S_y = +\hbar/2$	$S_x = +\hbar/2$	$\frac{1 + 1/\sqrt{2}}{4}$
$S_y = +\hbar/2$	$S_x = -\hbar/2$	$\frac{1 + 1/\sqrt{2}}{4}$
$S_y = -\hbar/2$	$S_x = +\hbar/2$	$\frac{1 - 1/\sqrt{2}}{4}$
$S_y = -\hbar/2$	$S_x = -\hbar/2$	$\frac{1 - 1/\sqrt{2}}{4}$.