Quantum Information: Homework 10

Due: 6 November 2018

1 Oracle construction

Consider functions that map a single bit to a single bit. There are four possible functions that do this:

$$f_1(x) = 0$$
$$f_2(x) = 1$$
$$f_3(x) = x$$
$$f_4(x) = 1 \oplus x$$

where x is a single bit. For each function, consider the standard oracle defined via its action on computational basis states as

$$\hat{U}_f \left|x\right\rangle \left|y\right\rangle := \left|x\right\rangle \left|y \oplus f(x)\right\rangle .$$

a) Construct the matrices that represent the oracle for each possible function.

b) For each function, indicate how the oracle can be constructed from CNOT and single qubit gates.

2 Single bit function classification: Deutsch algorithm

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where x is a single bit. For each function, consider the standard oracle defined via its action on computational basis states as

$$\hat{U}_f \left|x\right\rangle \left|y\right\rangle := \left|x\right\rangle \left|y \oplus f(x)\right\rangle .$$

a) Describe whether each function is constant (over all possible inputs) or not.

b) Suppose that you are provided with the oracle for one of these functions but do not know which function. Provide a classical procedure, using the oracle and computational basis states only, that can determine whether the chosen function is constant or not. How many oracle queries does this require?

c) Now suppose that the following state is used as an input to the oracle:

$$\frac{1}{2}\Big(|0\rangle + |1\rangle\Big)\Big(|0\rangle - |1\rangle\Big).$$

Determine the state of the after the oracle query (the state will depend on the function and can be expressed in terms of $f(x)$). Would a computational basis measurement on the argument register determine whether the function is constant or not?

d) Suppose that the input to the oracle is

$$\frac{1}{2}\Big(|0\rangle + |1\rangle\Big)\Big(|0\rangle - |1\rangle\Big)$$

and that immediately after the oracle a Hadamard is applied to the argument register. Determine the state of the two qubits after this. Evaluate the state of the argument register for all possible choices of oracle function. Would a measurement on the argument register reveal whether the function was constant or not? How many oracle queries were required to accomplish this?

3 Deutsch-Jozsa algorithm

Consider the Deutsch-Jozsa problem for functions of three bits.

a) The following are candidates for balanced functions:

$$\begin{aligned} f_1(x_2, x_1, x_0) &= x_2 \\ f_2(x_2, x_1, x_0) &= x_2 \oplus x_1 \\ f_3(x_2, x_1, x_0) &= x_2 x_1 \oplus x_0 \\ f_4(x_2, x_1, x_0) &= x_2 x_1 \oplus x_2 \oplus x_1 \oplus x_0 \end{aligned}$$

Verify that each of these is a balanced function.

b) Consider the standard algorithm presented in class. For each function, determine the state of the argument register immediately prior to measurement. Would the algorithm be able to determine with certainty, which of these four functions was supplied?

4 IBM Q Experience

We will continue using the IBM Q Experience.

Go to a simulator at:

https://quantumexperience.ng.bluemix.net/qx/experience

Construct the circuit for all four possible functions that the Deutsch algorithm considers. Verify that the simulator produces the correct output and can distinguish between constant and npn-constant functions.