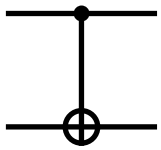
Quantum Information: Homework 9

Due: 30 October 2018

1 Bell state construction

Consider a pair of qubits that is subjected to the illustrated gate.



The four Bell states are:

$$|\Phi_+\rangle := \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$
$$|\Phi_-\rangle := \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$
$$|\Psi_+\rangle := \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$
$$|\Psi_-\rangle := \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right).$$

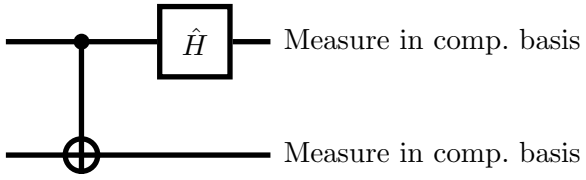
a) Verify that if the input state is $\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|0\rangle$, one of the Bell states is produced.

b) Now suppose that you aim to construct this Bell states using the input $|00\rangle$. Indicate how to modify the circuit using a single additional single qubit gate to accomplish this.

c) Indicate how to modify the circuit to construct any Bell state by starting with the input $|00\rangle$.

2 Bell state measurement

Consider a pair of qubits that is subjected to the illustrated gate. After the gate each is measured in the computational basis, $\{|0\rangle, |1\rangle\}$.



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a) Suppose that $|\Phi_+\rangle$ is input at the left. Determine the outcomes of the measurements and the probability with which they occur. Repeat this for the other three Bell states.

b) Collect the outcomes in a table, indicating the input Bell state and the pair of measurement outcomes for each. Does this entire operation form a measurement in the Bell basis?

3 Fanout gate

A classical fanout produces a copy of an input state

$$(x, 0) \mapsto (x, x)$$

a) Show how to use a Toffoli gate to construct a fanout gate.

b) Can this gate be used to copy an aribitrary input state? Explain your answer.

4 Rieffel, *Quantum Computing*, 6.2, page 121.

5 IBM Q Experience

We will continue using the IBM Q Experience.

a) Go to a simulator at:
https://quantumexperience.ng.bluemix.net/qx/experience
Construct a CNOT gate. Test the CNOT for all possible computational basis inputs. Print the output.

b) Construct the Bell state

$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

in the simulator and perform a computational basis measurement. Print the output.

c) Construct a Bell state measurement in the simulator (see an earlier problem). Input the Bell state

$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

and perform the measurement. Print the output.