Phys 396
Fall 2018

Quantum Information: Homework 8

Due: 23 October 2018

1 Hadamard gate

Consider the Hadamard gate

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

a) Show that

$$\hat{H} = \frac{1}{\sqrt{2}} \left(\hat{\sigma}_x + \hat{\sigma}_z \right).$$

b) Show that

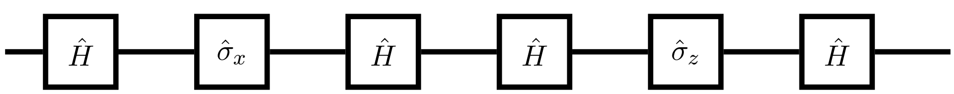
$$\begin{aligned} \hat{H}\hat{\sigma}_x\hat{H} &= \hat{\sigma}_z \\ \hat{H}\hat{\sigma}_y\hat{H} &= -\hat{\sigma}_y \\ \hat{H}\hat{\sigma}_z\hat{H} &= \hat{\sigma}_x \end{aligned}$$

c) Use these to show that

$$\begin{aligned} \hat{H}\hat{\sigma}_x &= \hat{\sigma}_z\hat{H} \\ \hat{H}\hat{\sigma}_y &= -\hat{\sigma}_y\hat{H} \\ \hat{H}\hat{\sigma}_z &= \hat{\sigma}_x\hat{H} \end{aligned}$$

Hint: First show that $\hat{H}^2 = \hat{I}$.

d) Use these to simplify the following circuit



2 Decomposition of single qubit gates

Define the rotation about x gate and the rotation about z gates as

$$\begin{aligned} \hat{R}_x(\theta) &:= e^{-i\theta\hat{\sigma}_x/2} \\ \hat{R}_z(\phi) &:= e^{-i\phi\hat{\sigma}_z/2} \end{aligned}$$

a) Determine an expression for

$$\hat{R}_x(\theta)\hat{R}_z(\phi)\hat{R}_x(\varphi).$$

in terms of Pauli operators.

b) Use this expression to show that

$$\hat{H} = ie^{i\alpha}\hat{R}_x(\pi/2)\hat{R}_z(\pi/2)\hat{R}_x(\pi/2)$$

for some real phase α. This gives a decomposition of the Hadamard gate in terms of basic rotations.

3 Controlled gates

Let $\hat{U}$ be unitary. Consider the general controlled $\hat{U}$ gate

$$\hat{C}_{\hat{U}} := |0\rangle\langle 0| \otimes \hat{I} + |1\rangle\langle 1| \otimes \hat{U}.$$

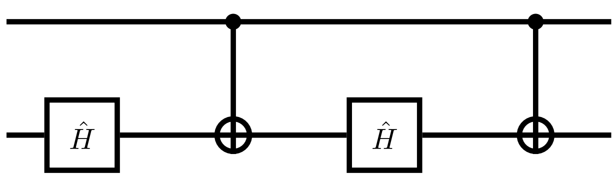
Note that the CNOT gate is exactly $\hat{C}_{\hat{\sigma}_x}$.

a) Determine matrix representations for $\hat{C}_{\hat{\sigma}_y}$ and $\hat{C}_{\hat{\sigma}_z}$.

b) Show that for any $\hat{U}$, the controlled gate $\hat{C}_{\hat{U}}$ is unitary.

c) Show that $\hat{C}_{\hat{\sigma}_y}\hat{C}_{\hat{\sigma}_z} = \hat{C}_{\hat{U}}$ for some $\hat{U}$. Provide $\hat{U}$.

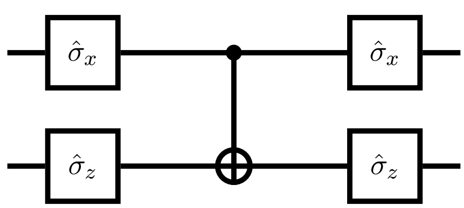
d) Reduce the following circuit to a single controlled gate.



e) Determine the single operator equivalent the following circuit. Your answer could be in the form of a matrix (more work),an operator of the form

$$|0\rangle\langle 0| \otimes \hat{A} + |1\rangle\langle 1| \otimes \hat{B},$$

or by listing how it maps $|00\rangle, |01\rangle, \ldots$.



4 Rieffel, *Quantum Computing*, 5.9, page 95.

5 Rieffel, *Quantum Computing*, 5.10, page 95.

6 Rieffel, *Quantum Computing*, 5.13, page 96.

7 IBM Q Experience

We will aim to runs some simple quantum information processing tasks on the IBM cloud quantum computer. The portal to this is IBM Q Experience. It allows anyone to submit quantum information processing tasks to one of their small scale quantum computing devices or to one of their simulators. In order to use this you will need to accumulate some of their "units." To start the process do the following.

a) Go to the IBM Q Experience website (link on course page) and register yourself.

b) In order to accumulate "units" you need to read the user guide. Access by clicking on the "Learn" tab and go to "Full User Guide." Read "Introducing the IBM Q Experience" and "The Weird and Wonderful World of the Qubit." Check your units in your profile. Printout a screenshot of this and attach it to the assignment.

c) The actual simulators and computers are located via:

   `https://quantumexperience.ng.bluemix.net/qx/experience`

   I encourage you to explore using this and generating circuits with measurements - but don't waste your "units" yet – use one of their simulators.