Phys 396
Fall 2018

Quantum Information: Homework 6

Due: 2 October 2018

1 Reiffel, *Quantum Computing*, 4.1, page 66. Start by expressing each bar and ket as a row or column vector. For two qubit systems use

$$|00\rangle \leftrightsquigarrow \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad |01\rangle \leftrightsquigarrow \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \qquad |10\rangle \leftrightsquigarrow \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \qquad |11\rangle \leftrightsquigarrow \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

2 Measurement operators

Suppose that a single qubit is measured in the basis

$$\left\{ \frac{1}{\sqrt{5}} |0\rangle + \frac{2i}{\sqrt{5}} |1\rangle , \frac{2}{\sqrt{5}} |0\rangle - \frac{i}{\sqrt{5}} |1\rangle \right\}.$$

a) Construct the two projector operators.
b) Verify that $\sum \hat{P}_i = \hat{I}$.
c) Verify that $\hat{P}_i^2 = \hat{P}_i$ for each projector.
d) Verify that $\hat{P}_i \hat{P}_j = 0$ if $i \neq j$.

3 Single qubit projectors and measurements

A single qubit is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle$$

prior to a measurement.

a) Suppose that the qubit is measured in the basis

$$\left\{ \frac{1}{\sqrt{2}} |0\rangle + \frac{i}{\sqrt{2}} |1\rangle , \frac{1}{\sqrt{2}} |0\rangle - \frac{i}{\sqrt{2}} |1\rangle \right\}$$

Construct the projectors for the measurement and use these to calculate the probability of each measurement outcome and the states after each measurement outcome.

b) Suppose that the qubit is measured in the basis

$$\left\{ \frac{1}{\sqrt{5}} |0\rangle + \frac{2i}{\sqrt{5}} |1\rangle , \frac{2}{\sqrt{5}} |0\rangle - \frac{i}{\sqrt{5}} |1\rangle \right\}.$$

Construct the projectors for the measurement and use these to calculate the probability of each measurement outcome.

4 Multiple qubit projector operators

Consider two qubits initially in the state

$$|\Psi\rangle = \frac{1}{\sqrt{3}}\,|00\rangle + \frac{1}{\sqrt{3}}\,|01\rangle + \frac{1}{\sqrt{3}}\,|10\rangle\,.$$

The left qubit is measured in the basis

$$\{|0\rangle\,, |1\rangle\}$$

and the right qubit is measured in the basis

$$\left\{\frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle\,, \frac{1}{\sqrt{2}}\,|0\rangle - \frac{1}{\sqrt{2}}\,|1\rangle\right\}$$

a) List the four combinations of measurement outcomes and the matrix that represents the measurement projector operator for each.

b) Use the projectors to determine the probabilities with which the various measurement outcomes occur and the states after each.

c) Suppose that only the left qubit is measured in the basis $\{|0\rangle\,, |1\rangle\}$ and the right qubit is ignored. Determine the probabilities of the two outcomes and the state of the system after each outcome.

5 Tensor products

Define the following matrices (the notation will be apparent later):

$$\hat{\sigma}_x := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\sigma}_z := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \hat{H} := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

a) Determine the matrix for $\hat{\sigma}_x \otimes \hat{\sigma}_x$.

b) Determine the matrix for $\hat{\sigma}_x \otimes \hat{\sigma}_z$.

c) Determine the matrix for $\hat{\sigma}_x \otimes \hat{H}$.

d) Determine the matrix for $\hat{H} \otimes \hat{\sigma}_x$.

e) Determine the matrix for $\hat{H} \otimes \hat{H}$.

6 News item

Find a news item published within the last year that describes an advance in quantum computing or quantum information. Post a link to the item in the discussion thread for 2 October. Summarize (here) in a single paragraph what the article describes.