Quantum Information: Homework 5

Due: 25 September 2018

1 Product states

Suppose that the states for one qubit are

$$|\phi_1\rangle := \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$
$$|\phi_2\rangle := \frac{1}{\sqrt{5}}\left(|0\rangle + i\,|1\rangle\right)$$

and for another qubit are

$$|\chi_1\rangle := \frac{1}{\sqrt{5}}\left(|0\rangle - 2\,|1\rangle\right)$$
$$|\chi_2\rangle := \frac{1}{\sqrt{5}}\left(|0\rangle - 2i\,|1\rangle\right).$$

a) Express the states $|\Psi_1\rangle := |\phi_1\rangle\,|\chi_1\rangle$ and $|\Psi_2\rangle := |\phi_2\rangle\,|\chi_2\rangle$ in terms of the basis

$$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}.$$

b) Evaluate $\langle\Psi_1|\Psi_2\rangle$.

2 Bell basis

The Bell states are

$$|\Phi_+\rangle := \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$
$$|\Phi_-\rangle := \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$
$$|\Psi_+\rangle := \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$
$$|\Psi_-\rangle := \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right).$$

a) Show that these are orthonormal.

b) Consider a generic state

$$|\chi\rangle = a_0\,|00\rangle + a_1\,|01\rangle + a_2\,|10\rangle + a_3\,|11\rangle.$$

Express this state as a superposition of the Bell states. *This shows that the Bell states form an orthonormal basis.*

3 Entangled and product states

For each of the following describe whether the state is entangled or a product state. If it is a product state, describe the pair of measurements that would yield one particular outcome with certainty.

a) $|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|00\rangle + |01\rangle\right).$

b) $|\Psi\rangle = \frac{1}{\sqrt{3}} \left(|00\rangle + |01\rangle + |11\rangle\right).$

c) $|\Psi\rangle = \frac{1}{2} \left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right).$

d) $|\Psi\rangle = \frac{1}{2} \left(|00\rangle - i\,|01\rangle + i\,|10\rangle + |11\rangle\right).$

4 Measurements on two qubits

Suppose that two qubits are in the state

$$|\Psi\rangle := \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle\right).$$

a) Determine whether this state is entangled.

b) Suppose that each qubit is measured in the basis $\{|0\rangle, |1\rangle\}$. List all outcomes and their probabilities.

c) Suppose that each qubit is measured in the basis

$$\left\{ \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle\right), \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle\right) \right\}.$$

List all outcomes and their probabilities.

d) Suppose that each qubit is measured in the basis

$$\left\{ \frac{1}{\sqrt{2}} \left(|0\rangle + i\,|1\rangle\right), \frac{1}{\sqrt{2}} \left(|0\rangle - i\,|1\rangle\right) \right\}.$$

List all outcomes and their probabilities.

5 Two spin-1/2 particles

Consider two spin-1/2 particles in one of the following entangled states:

$$|\Psi_-\rangle := \frac{1}{\sqrt{2}} \left(|+\hat{\boldsymbol{z}}\rangle\,|-\hat{\boldsymbol{z}}\rangle - |-\hat{\boldsymbol{z}}\rangle\,|+\hat{\boldsymbol{z}}\rangle\right)$$

$$|\Phi_+\rangle := \frac{1}{\sqrt{2}} \left(|+\hat{\boldsymbol{z}}\rangle\,|+\hat{\boldsymbol{z}}\rangle + |-\hat{\boldsymbol{z}}\rangle\,|-\hat{\boldsymbol{z}}\rangle\right).$$

We aim to consider measurements of any spin components on each particle.

a) First start with the usual representation of $\{|+\hat{\boldsymbol{n}}\rangle, |-\hat{\boldsymbol{n}}\rangle\}$ to show that

$$|+\hat{\boldsymbol{z}}\rangle = \cos\left(\frac{\theta}{2}\right)|+\hat{\boldsymbol{n}}\rangle + \sin\left(\frac{\theta}{2}\right)|-\hat{\boldsymbol{n}}\rangle$$
$$|-\hat{\boldsymbol{z}}\rangle = e^{-i\phi}\sin\left(\frac{\theta}{2}\right)|+\hat{\boldsymbol{n}}\rangle - e^{-i\phi}\cos\left(\frac{\theta}{2}\right)|-\hat{\boldsymbol{n}}\rangle\,.$$

b) Express $|\Psi_-\rangle$ and $|\Phi_+\rangle$ in terms of $\{|+\hat{\boldsymbol{n}}\rangle, |-\hat{\boldsymbol{n}}\rangle\}$.

c) Suppose that both particles are in the state $|\Psi_-\rangle$ and each is subjected to a SG $\hat{\boldsymbol{n}}$ measurement. List all possible pairs of outcomes and the probabilities with which they occur. Are the outcomes correlated?

d) Suppose that both particles are in the state $|\Phi_+\rangle$ and each is subjected to a SG $\hat{\boldsymbol{n}}$ measurement. List all possible pairs of outcomes and the probabilities with which they occur. Do these depend on the angle ϕ associated with $\hat{\mathbf{n}}$?

e) Given that the state of the particles is $|\Phi_+\rangle$ describe the choices of SG measurements such that there two outcomes will be the same with certainty.

f) Given that the state of the particles is $|\Phi_+\rangle$ describe the choices of SG measurements such that there two outcomes will be the opposite with certainty. Are the outcomes correlated with certainty for all possible SG measurement choices?

g) Suppose that only one of these particles is subjected to an SG $\hat{\boldsymbol{n}}$ measurement and the other is disregarded. List the outcomes and probabilities of measurement for this single particle. Do these depend on the choice of measurement? Is there any way that one can choose an SG measurement on just one of the particles so that one outcome is more likely than the other?

h) Given that one has just one of the pair of particles and the pair is known to be in either state $|\Psi_-\rangle$ or $|\Phi_+\rangle$, can one determine in which state the pair is by doing an SG measurement on the single particle?

i) Suppose that the two particles are in state $|\Psi_-\rangle$ and a SG $\hat{\boldsymbol{x}}$ measurement is performed on one while a SG $\hat{\boldsymbol{z}}$ measurement is performed on the other. List all possible pairs of outcomes and the probabilities with which they occur. Are the outcomes correlated?

6 News item questions

Review the news items that have been posted to the D2L discussion forums and provide two questions about their content. List these on your assignment and post these to the discussion thread for 25 September.